The Energy Harvesting Two-Way Decode-and-Forward Relay Channel with Stochastic Data Arrivals

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***Abstract*—In this paper, a two-way relay channel is considered with energy harvesting nodes and stochastic data arrivals at the source nodes. The batteries and data buffers at all nodes are of finite storage capability. The sum throughput maximization problem for this set up is shown to be a convex optimization problem, and the optimal offline policy is found. Numerical results are presented to demonstrate the optimal policy for different channel setups, and its performance.**

***Index Terms*—Energy harvesting, two-way relay channel, throughput maximization, decode-and-forward, finite battery/buffer.**

I. Introduction

Efficient management of available energy is an important issue for wireless networks in general. This issue becomes even more prominent in energy harvesting wireless networks where nodes *harvest* energy in an intermittent fashion from external sources, such as solar cells, wind mills, water mills, and piezoelectric devices. In this work, we study throughput maximization in a two-way relay channel with energy harvesting. The two-way nature of the communication scheme considered in this work, along with the energy harvesting property of all nodes translates the problem into a more challenging one; it also yields ample insights into how a wireless network can utilize intermittent energy in the most efficient fashion.

In recent years, energy harvesting wireless networks have attracted a lot of attention and various channels models have been studied in an energy harvesting setup. The single user channel is studied in [1] with an energy harvesting source node that can also receive packets of data to transmit. These stochastic data arrivals are observed to make the throughput maximization problem more interesting, and also more challenging. Reference [2] has studied the same model with a finite capacity battery and found the throughput maximizing policy. Reference [3] has extended this model to fading channels and solved the throughput maximization problem using directional water-filling. Energy harvesting has also been studied in multi-user settings, such as the multiple access and broadcast channels studied in [4], [5], and relay channels [6]–[11]. The two-hop channel is studied in [8], where the solution is found for two energy harvests at the source, and in [9], where the effect of the buffer size at the relay is analyzed. Finally, the two-way relay channel, which this work focuses on, is studied in reference [10] where the optimal policy is found to maximize throughput with a decode-and-forward relay, and also in [11] where the generalized directional water-filling solution is found and it is shown that hybrid relaying strategies result in higher throughput. This work mainly differs from [10] and [11] in that the source nodes can receive data to transmit during the course of communication, and the relay employs *finite-sized* data buffers so that it does not have to forward incoming data immediately after it is received. In this sense, a new optimization dimension is introduced, i.e., the optimal transmission policy must also allocate the data in the source and relay buffers in addition to transmit powers properly to achieve maximum throughput.

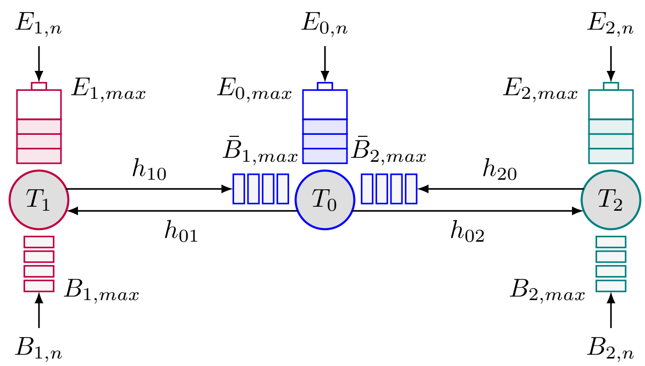

Fig. 1: The two-way relay channel with energy harvesting nodes and stochastic data arrivals.

In this paper, we solve the throughput maximization problem for an energy harvesting two-way relay channel with stochastic data arrivals and a half-duplex decode-and-forward relay in an offline setting where all arrivals are known non-causally. We impose finite-battery and finite-buffer constraints at all nodes to gain more realistic insights and analyze how these size constraints affect the optimal policy. We show that the throughput maximization problem can be expressed as a convex optimization problem, which we solve numerically. We observe that optimal transmission policies for the two-way relay channel do not always follow those for simpler channel models, e.g., the single user channel studied in [1], [2], and that higher throughput can be achieved by employing larger buffers at the relay, but this improvement is limited as throughput saturates after a certain buffer size.

II. System Model

Consider a Gaussian two-way relay channel with two source nodes, T_1 and T_2 and a decode-and-forward relay node, T_0. The two source nodes wish to communicate with each other, and can do so only using the intermediate relay. The relay is a half-duplex node, and communication takes place in two phases: phase I while the relay listens, and phase II while the relay transmits. While the relay listens, the channel gains from T_1 to T_0 and from T_2 to T_0 are denoted by h_{10} and h_{20}, respectively. Similarly, h_{01} and h_{02} denote the channel gains from T_0 to T_1 and from T_0 to T_2 while the relay transmits. The additive white Gaussian noise at node T_j has zero mean and unit variance, for $j = 0, 1, 2$, see Fig. 1.

All nodes in our model receive energy intermittently. The source nodes T_1 and T_2 receive data that they have to send to each other intermittently. We refer to the time duration between any two consecutive energy or data arrivals as an epoch. N denotes the total number of epochs by deadline T, s_n denotes the beginning of the nth epoch and we set $s_{N+1} = T$. The length of the nth epoch is denoted by $l_n = s_{n+1} - s_n$, $n = 1, 2, \ldots, N$. Node T_j harvests $E_{j,n}$ units of energy at the beginning of the nth epoch, s_n, for $j = 0, 1, 2$. Also, the source node T_j receives $B_{j,n}$ bits of data at s_n, for $j = 1, 2$. If node T_j does not harvest any energy (resp. data) at s_n for some j and n, then we set $E_{j,n} = 0$ (resp. $B_{j,n} = 0$). Node T_j can store up to $E_{j,max}$ units of energy in its battery, $j = 0, 1, 2$. Source node T_j, $j = 1, 2$, can store up to $B_{j,max}$ bits in its data buffer. The relay node is given two finite-capacity buffers, with $\bar{B}_{1,max}$ and $\bar{B}_{2,max}$ bits respectively, to store incoming data from nodes T_1 and T_2. All arrival instants s_n, energy amounts $E_{j,n}$ and data amounts $B_{j,n}$ are assumed to be known non-causally as in [1]–[11].

Define $\Delta_n \in [0, 1]$ to denote the fraction of the nth epoch during which the sources transmit and the relay listens. Then, the lengths of phase I and phase II in the nth epoch are given by $\Delta_n l_n$ and $(1 - \Delta_n) l_n$, respectively. The first phase of the two-way communication scheme amounts to a multiple-access channel with two transmitters, T_1 and T_2, and a receiver, T_0, leading to rates [12]

$$R_k \leq C(h_{k0}^2 P_k), \quad k = 1, 2, \tag{1a}$$

$$R_1 + R_2 \leq C(h_{10}^2 P_1 + h_{20}^2 P_2), \tag{1b}$$

where $C(x) \triangleq \frac{1}{2}\log(1 + x)$, R_1 and R_2 denote the rates achieved from T_1 to T_0 and from T_2 to T_0, respectively, and P_k denotes the transmit power at node T_k, $k = 1, 2$. The second phase is a broadcast channel with side information. Then, using the coding scheme in [10] where the relay broadcasts a function of the two codewords that correspond to the messages decoded at T_0 in phase I, one can achieve

$$R_1 \leq C(h_{02}^2 P_0), \quad R_2 \leq C(h_{01}^2 P_0), \tag{2}$$

where P_0 denotes the transmit power at the relay and R_k denotes the rate of node T_k's message that is to be received at node $T_{\bar{k}}$, $k, \bar{k} = 1, 2$, $k \neq \bar{k}$. Since the achievable rates in (1) and (2) are concave in transmit powers, by a similar argument to that in [1, Lemma 2], we can conclude that the transmit power at each node should remain constant throughout an epoch while the node is transmitting. We denote the average transmit power at node T_j in the nth epoch by $p_{j,n}$, $j = 0, 1, 2$. The transmit power is averaged over the duration of the epoch, i.e., node T_k, $(k = 1, 2)$ transmits with power $p_{k,n}/\Delta_n$ for $\Delta_n l_n$ seconds and T_0 transmits with power $p_{0,n}/(1 - \Delta_n)$ for $(1 - \Delta_n) l_n$ seconds in the nth epoch. In addition, we denote by $r_{k,n}^I$ the average rate achieved from T_k to T_0 in phase I of the nth epoch, and by $r_{k,n}^{II}$ the rate of node T_k's message that the relay forwards in phase II of the nth epoch for $k = 1, 2$. These rates yield the amount of data each node transmits, e.g., T_k transmits $l_n r_{k,n}^I$ bits in phase I of the nth epoch, $k = 1, 2$.

III. Throughput Maximization

In this section, we express and solve the throughput maximization problem for the two-way relay channel. We begin by describing the feasibility conditions that have to be satisfied by all policies. Since all nodes in the channel model are energy harvesting and have finite-capacity batteries, the amount of energy in the battery at s_n has to be sufficient for transmission in the nth epoch and cannot be greater than the battery's capacity, i.e.,

$$l_n p_{j,n} \leq \sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n-1} l_i p_{j,i} \leq E_{j,max}. \tag{3}$$

The stochastic data arrivals introduce similar constraints. The amount of data a source node can transmit in the nth epoch is upperbounded by the amount of data in its buffer at s_n, which is in turn upper-bounded by the size of the buffer, i.e.,

$$l_n r_{k,n}^I \leq \sum_{i=1}^{n} B_{k,i} - \sum_{i=1}^{n-1} l_i r_{k,i}^I \leq B_{k,max}. \tag{4}$$

The relay also cannot forward messages before it receives them and the two buffers at the relay have finite capacities. Thus,

$$0 \leq \sum_{i=1}^{n} l_n (r_{k,i}^I - r_{k,i}^{II}) \leq \bar{B}_{k,max}. \tag{5}$$

Lastly, the rates that are achieved in each epoch are constrained by the rate regions in (1) and (2) as follows.

$$r_{k,n}^I \leq \Delta_n C(h_{k0}^2 p_{k,n}/\Delta_n), \tag{6}$$

$$r_{1,n}^I + r_{2,n}^I \leq \Delta_n C((h_{10}^2 p_{1,n} + h_{20}^2 p_{2,n})/\Delta_n), \tag{7}$$

$$r_{k,n}^{II} \leq (1 - \Delta_n) C(h_{0\bar{k}}^2 p_{0,n}/(1 - \Delta_n)), \tag{8}$$

where $k, \bar{k} = 1, 2$, $k \neq \bar{k}$. With all the constraints given above, the optimization problem that maximizes the total amount of data that the relay forwards can be written as

$$\max_{p_{j,n}, r_{k,n}^I, r_{k,n}^{II}, \Delta_n} \sum_{n=1}^{N} l_n (r_{1,n}^{II} + r_{2,n}^{II}) \tag{9a}$$

$$\text{s.t. } p_{j,n}, r_{k,n}^I, r_{k,n}^{II} \geq 0, \ 0 \leq \Delta_n \leq 1, \text{ and (3)–(8)}, \tag{9b}$$

for all $n = 1, 2, \ldots, N$, $j = 0, 1, 2$, $k, \bar{k} = 1, 2$, $k \neq \bar{k}$. The constraints in (3)–(5) are linear in the optimization parameters. The constraints in (6)–(8) do not violate convexity of the feasible region since the right-hand sides in (6)–(8) are of the form $yC(x/y)$ for some x and y, which is jointly concave in x and y as is the perspective of a concave function, $C(x)$ [13, §3.2.6]. Hence, (9) is a convex optimization problem and can be solved using available algorithms in convex programming.

Note that it is necessary to leave $r_{k,n}^I$ and $r_{k,n}^{II}$ as optimization parameters in (9) instead of expressing them in terms of $p_{j,n}$ and Δ_n, contrary to previous work on energy harvesting, e.g., [1], [2], [10]. Due to the two-way nature of the channel model, the nodes may not always be able to fully exploit their transmit powers. As an example, suppose the relay has messages only from T_1 in an epoch. Then, $r_{2,N}^{II} = 0$ since the relay does not have any messages from T_2 to forward, but it still has to spend some energy to forward node T_1's messages. Thus, it uses its energy to forward messages only in one direction, although it would have been possible to forward in both directions had the relay had any messages from T_2.

We solve the constrained convex optimization problem in (9) using the method of steepest descent with random epoch lengths, energy arrivals and data arrivals. Since the objective in (9) is not strictly concave, it is possible to have multiple optimal solutions. However, the steepest descent algorithm is guaranteed to converge to one of these solutions that will yield the optimum sum throughput since the problem is convex.

Remark 1: Our solution of (9) is valid for all special cases of the model in Section II, some of which are listed below.

Two-way relay channel with no buffers at the relay and infinite backlog at the sources. Also studied in [10], [11], the optimal policy for this channel requires that the relay forward the same amount of data as it receives in each epoch, and can be found by solving (9) with $B_{k,max} = \infty$, $\bar{B}_{k,max} = 0$, $B_{k,n} = 0$, $k = 1, 2$, $n = 2, 3, \ldots, N$ and $B_{1,1} = B_{2,1} = \infty$.

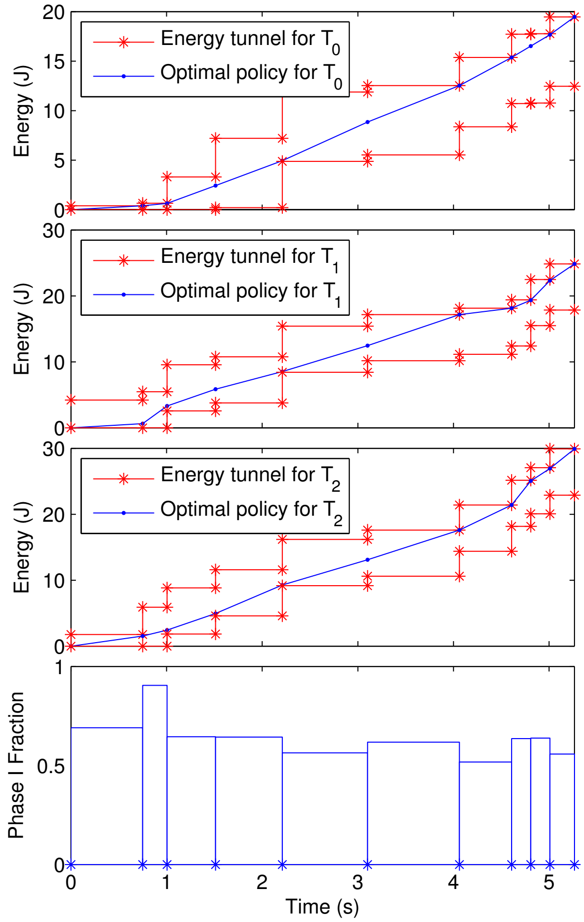

Fig. 2: Optimal transmission policy for an energy harvesting two-way relay channel.

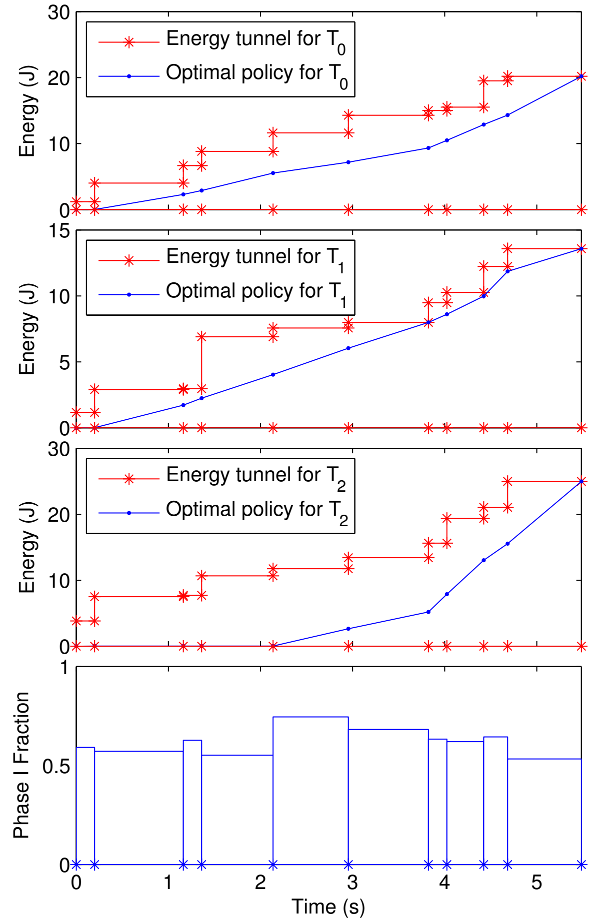

Fig. 3: Optimal transmission policy for an energy harvesting two-way relay channel with delayed data arrivals.

Two-hop channel. This channel model is a special case of our model with data flowing in only one direction. Therefore, by setting $E_{2,n} = 0, n = 1, 2, \dots, N$, we can prevent node T_2 from transmitting any data to T_0 and find the optimal policy for this model by solving (9). This model is studied in [8] and [9] with an infinite backlog at T_1 instead of data arrivals.

Two-way channel. The optimal policy for the two-way channel with stochastic data arrivals at the two sources can be found by solving (9) with $h_{20} = h_{02} = \infty$. With these selections, nodes T_0 and T_2 are merged together to form a two-way channel.

Single user channel. This channel model with data arrivals is studied in [1]. The optimal policy can also be found by setting $h_{20} = h_{02} = \infty, \bar{B}_{1,max} = \infty, E_{2,n} = 0, n = 1, 2, \dots, N$ and solving (9). This way, T_0 and T_2 are merged to reduce the model into a one-hop channel. Also, T_2 is not allowed to transmit which results in a single user channel.

IV. Numerical Results

In this section, we present numerical examples of the solution of the sum throughput maximization problem in (9), and demonstrate the resulting transmission policies.

Fig. 2 shows the optimal policy that maximizes throughput in a setting with unit channel gains, $B_{k,max} = 2$ kbit, $\bar{B}_{k,max} = 1$ kbit, $k = 1, 2$, and $E_{j,max} = 7$ J, $j = 0, 1, 2$. The upper wall of the energy tunnel represents the cumulative harvested energy and the lower wall represents how much of this energy would have to be wasted, if not utilized, due to the finite capacity of the batteries. As can be seen, the optimal policy is not necessarily the shortest path between the beginning and the end of the tunnels unlike previous work, e.g., [1], [2]. This is due to the fact that the optimal policy must consider how the energy harvesting profiles at all nodes should interact for optimality. For example, node T_1 does not choose the shortest path for the first two epochs, but rather chooses to transmit with less power in the first epoch. This is because the other two nodes T_0 and T_2 harvest low energy at s_1, and thus cannot transmit with high power. In order for the relay to utilize its low energy in the first epoch, T_1 also lowers its power. But, now T_1 has more energy to spend in the second epoch which will otherwise be wasted since $E_{1,3}$ almost fills its battery. In order to utilize this extra energy at T_1 efficiently, Δ_2 is increased to almost 1, which could possibly cause the relay to drop some of the packets if it did not have data buffers.

Fig. 3 shows the optimal policy for the same setup, except with infinite-capacity batteries and delayed data arrivals at the source nodes. Nodes T_1 and T_2 do not spend any energy while they do not have any data. The relay chooses to forward only T_1's messages in epochs 2, 3, and 4, instead of saving its energy until s_5 which

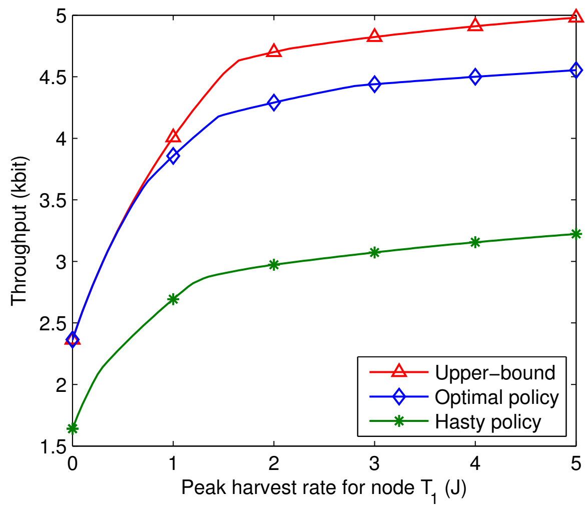

Fig. 4: Throughput achieved along with lower- and upper-bounds for varying peak harvest rates for node T_1.

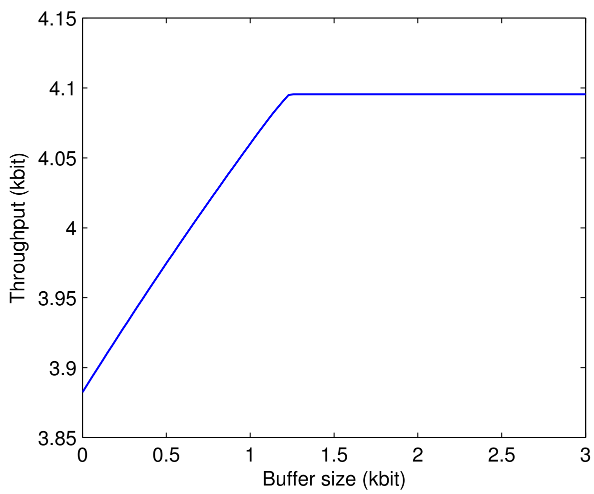

Fig. 5: Throughput achieved for varying buffer sizes at T_0.

would be suboptimal since the relay has sufficient energy for later epochs and higher rates are achievable if communication takes place only in one direction.

Fig. 4 shows the throughput achieved by the optimal policy, along with lower- and upper-bounds in a setting with unit channel gains, $B_{k,max} = 10$ kbit, $\bar{B}_{k,max} = 1$ kbit, $k = 1, 2$, and $E_{j,max} = 5$ J, $j = 0, 1, 2$. The peak energy harvesting rate is fixed at 5 J for T_0 and T_2, and varied from 0 J to 5 J for T_1. The upper-bound is the case where all nodes receive all the energy at s_1 with infinite batteries, so they are able to expend their energy more liberally. The lower-bound is the hasty policy where the nodes do not have batteries, and the relay does not have data buffers. The throughput curve achieved by the optimal policy is monotonically increasing and concave in the peak harvest rate for node T_1. It is observed that batteries and data buffers can help achieve higher performance in an energy harvesting network. The achieved throughput is close to the upper-bound under energy deficient conditions which are more likely to occur in an energy harvesting setup.

Lastly, Fig. 5 shows how the buffer size at the relay affects the achievable throughput. $\bar{B}_{1,max}$ and $\bar{B}_{2,max}$ are set equal, and varied from 0 kbit, the case in [10] and [11], to 3 kbit. As can be seen, larger buffers allow the relay to store more data for later epochs when the relay can more efficiently forward messages, and naturally, the achieved throughput increases in buffer size. However, after a certain buffer size, the achieved throughput is constant since at this critical point, the relay has sufficient storage for optimality and larger buffers are redundant.

V. Discussion and Conclusion

In this paper, we studied a two-way relay channel with finite batteries, finite buffers, and stochastic data arrivals at the source nodes. We formulated the throughput maximization problem as a convex optimization problem and solved it for the general system model. We presented numerical results to show optimal policy examples and performance evaluations of our solution. It should be noted that relaying schemes other than decode-and-forward can also be considered for a two-way relay channel with stochastic data arrivals. For example, the lattice-forwarding scheme in [14] is shown to improve performance in [11] in an energy harvesting two-way relay channel with no buffers at the relay. Models with more elaborate multi-directional information flows such as the multi-pair and multi-way energy harvesting relay channels are left as future work.

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