Energy Harvesting Two-Way Communications with Limited Energy and Data Storage

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***Abstract*—In this paper, we study two-way communication scenarios with energy harvesting. In particular, we consider the two-way and two-way relay channels with finite data storage. We solve the throughput maximization problem with finite batteries and finite data buffers. This entails iteratively solving an energy problem, which distributes the available energy over the course of the communication session, and a data problem, which schedules how much data to send at each node over the course of the communication session. We provide a directional waterfilling interpretation to the energy problem with the addition of water pumps and overflow protection bins. The data problem turns out to be a linear program which we solve by forward induction. We provide numerical results demonstrating the impact of the battery and buffer sizes on the achieved throughput. We observe, for communication scenarios of interest, that a relatively modest size of data storage is sufficient to harness the performance benefits of data buffering, i.e., to achieve the throughput values without buffer size limitations.**

I. Introduction

Energy harvesting nodes acquire their energy in an intermittent fashion over the course of their operation [1]. Energy harvesting communications has been studied in a variety of models in order to assess the impact of the harvested energy and its storage [1]–[10] on system performance. By contrast, although wireless networks also receive their data intermittently to a large extent, this aspect has not been studied in depth for energy harvesting communications.

Prior work in energy harvesting communications dates back to references [1] and [2] which study an energy harvesting transmitter, and identify the optimal transmission policies. References [3] and [4] derive waterfilling interpretations to the optimal transmission policies for the fading channel and the interference channel. Multi terminal networks including the two-hop network, the multiple access channel, and the two-way and multi-way relay channels are studied in [5]–[9]. Reference [10] proposes a framework for throughput maximization in a wireless network with energy harvesting transmitters and receivers, and energy storage limitations, by decoupling the throughput maximization problem into energy efficiency and energy harvesting adaptation problems. While these prior efforts assume infinite data storage at the nodes, reference [11] is particularly relevant to our work in that it has provided the first energy harvesting model with finite data buffers in the single user channel. In [11], the transmission completion time minimization problem for an energy harvesting transmitter is studied with finite data and energy storage, and data arrivals. The communication scenario in [11] does not allow dropped packets, an assumption we shall relax in this paper.

In this work, we study the throughput maximization problem for two-way energy harvesting communication systems with finite data buffers, extending our recent work on the single user channel [12] to a multi terminal scenario. We first study sum throughput maximization for the bidirectional channel composed of two energy harvesting half duplex transmitters. We consider intermittent data arrivals at the transmitters as well as intermittent energy arrivals. We solve the throughput maximization problem by first decomposing it into an energy problem and a data problem for each transmitter. We use alternating maximization between these sub-problems which converges to the optimal solution yielding the jointly optimal power and data policies for each node. We identify a directional waterfilling interpretation for the optimal solution for the energy problem, and propose a forward induction based solution for the data problem.

Next, we add an energy harvesting relay between the two transmitters, and solve the throughput maximization problem with finite energy and finite data storage constraints for the two-way relay channel. We show that this problem can also be decomposed into the same sub-problems. We provide numerical results demonstrating how the data buffer and battery capacities impact the optimal throughput. We observe that a modest size buffer is sufficient to achieve the optimal throughput.

II. Throughput Maximization for the Two-Way Channel

A. System Model

Consider an additive white Gaussian noise (AWGN) two-way channel with half duplex energy harvesting transmitters T_1 and T_2 as shown in Fig. 1. Without loss of generality, the AWGN variances and energy harvests are normalized such that the instantaneous rate achieved with transmit power p is $C(p) \triangleq \frac{1}{2}\log(1+p)$ in each direction. The time interval between any two consecutive energy or data arrivals is defined

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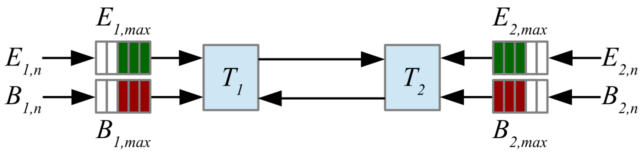

Fig. 1. The energy harvesting two-way channel with a finite battery and a finite data buffer at each transmitter.

to be an epoch. We consider communication with a deadline with N denoting the number of epochs by the deadline. In the nth epoch, node T_i harvests $E_{i,n}$ ($\leq E_{i,max}$) units of energy, and receives $B_{i,n}$ ($\leq B_{i,max}$) units of data at a constant rate. Here, $E_{i,max}$ and $B_{i,max}$ denote the size of the battery and the data buffer at node T_i, $i = 1, 2$.

B. Problem Statement and Solution

The throughput maximization problem for the half duplex two-way channel can be expressed as

$$\max_{\substack{\mathbf{p},\mathbf{w},\mathbf{r},\mathbf{d}\geq 0,\\ 0\leq\mathbf{\Delta}\leq 1}} \sum_{i=1}^{N} l_i(r_{1,i}+r_{2,i}) \tag{1a}$$

$$\text{s.t.} \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n} E_{j,i}, \tag{1b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq E_{j,max}, \tag{1c}$$

$$\sum_{i=1}^{n}(l_i r_{j,i}+d_{j,i}) \leq \sum_{i=1}^{n} B_{j,i}, \tag{1d}$$

$$\sum_{i=1}^{n} B_{j,i} - \sum_{i=1}^{n}(l_i r_{j,i}+d_{j,i}) \leq B_{j,max}, \tag{1e}$$

$$r_{1,n} \leq \Delta_n C(\frac{p_{1,n}}{\Delta_n}),\ r_{2,n} \leq (1-\Delta_n)C(\frac{p_{2,n}}{1-\Delta_n}), \tag{1f}$$

$$\forall j = 1, 2,\ n = 1, 2, \ldots, N, \tag{1g}$$

where $p_{j,n}$, $w_{j,n}$, $r_{j,n}$, and $d_{j,n}$ denote the transmit power, wasted energy, achieved rate, and dropped packets[1] for node T_j in the nth epoch, $j = 1, 2$, $n = 1, 2, \ldots, N$. Δ_n and $1-\Delta_n$ denote the fractions of the nth epoch reserved for node T_1's transmission and node T_2's transmission, respectively. Bold face notation is used to denote vectors of variables, e.g., $\mathbf{p} = (p_{1,1}, \ldots, p_{1,N}, p_{2,1}, \ldots, p_{2,N})$, and $\mathbf{\Delta} = (\Delta_1, \ldots, \Delta_N)$.

In (1), (1b) (cf. (1d)) and (1c) (cf. (1e)) express the energy (cf. data) causality and the energy (cf. data) storage constraints, respectively. Our approach to solve (1) is to decompose it into smaller problems and solve each one of those. In order to do this, we first define an inner problem over Δ_n as

$$\max_{0\leq\Delta_n\leq 1} \Delta_n C(\frac{p_{1,n}}{\Delta_n}) + (1-\Delta_n)C(\frac{p_{2,n}}{1-\Delta_n}). \tag{2}$$

[1]Unlike reference [11], we do not prohibit dropped packets for the purpose of achieving optimal end-to-end throughput.

Recalling that $C(p)$ is concave, we have that

$$\Delta_n C(\frac{p_{1,n}}{\Delta_n}) + (1-\Delta_n)C(\frac{p_{2,n}}{1-\Delta_n}) \leq C(p_{1,n}+p_{2,n}). \tag{3}$$

That is, $C(p_{1,n}+p_{2,n})$ is an upper-bound on the objective of (2) which can be achieved by setting $\Delta_n = \Delta_n^* \triangleq \frac{p_{1,n}}{p_{1,n}+p_{2,n}}$. Thus, we can find optimal Δ_n for all $n = 1, 2, \ldots, N$ given all the other decision variables. Let $C_{1,n}^*(p_{1,n}) = \Delta_n^* C(p_{1,n}/\Delta_n^*)$ and $C_{2,n}^*(p_{2,n}) = (1-\Delta_n^*)C(p_{2,n}/(1-\Delta_n^*))$. We have

$$\max_{\mathbf{p},\mathbf{w},\mathbf{r},\mathbf{d}\geq 0} \sum_{i=1}^{N} l_i(r_{1,i}+r_{2,i}) \tag{4a}$$

$$\text{s.t.} \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n} E_{j,i}, \tag{4b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq E_{j,max}, \tag{4c}$$

$$\sum_{i=1}^{n}(l_i r_{j,i}+d_{j,i}) \leq \sum_{i=1}^{n} B_{j,i}, \tag{4d}$$

$$\sum_{i=1}^{n} B_{j,i} - \sum_{i=1}^{n}(l_i r_{j,i}+d_{j,i}) \leq B_{j,max}, \tag{4e}$$

$$r_{j,n} \leq C_{j,n}^*(p_{j,n}),\ \forall j = 1, 2,\ n = 1, 2, \ldots, N. \tag{4f}$$

Let us now replace the objective of (4) with

$$\sum_{i=1}^{N} l_i(\min\{r_{1,i}, C_{1,i}^*(p_{1,i})\} + \min\{r_{2,i}, C_{2,i}^*(p_{2,i})\}). \tag{5}$$

This yields an equivalent problem where constraint (4f) is redundant. The remaining constraints (4b), (4c), (4d), and (4e) are separable between $(\mathbf{p}, \mathbf{w})$ and $(\mathbf{r}, \mathbf{d})$. Moreover, each constraint is separable between nodes T_1 and T_2. Thus, we can solve (4) using alternating maximization [13, §2.7]. We start with arbitrary feasible initial solutions $(\mathbf{p}_j^{[0]}, \mathbf{w}_j^{[0]})$ and $(\mathbf{r}_j^{[0]}, \mathbf{d}_j^{[0]})$, $j = 1, 2$ where the iteration index is given in the superscript and the user index is given in the subscript, e.g., $\mathbf{p}_1^{[0]} = (p_{1,1}^{[0]}, p_{1,2}^{[0]}, \ldots, p_{1,N}^{[0]})$. In order to converge to the optimal solution, we update the current solution by solving

$$(\mathbf{p}_j^{[k]}, \mathbf{w}_j^{[k]}) = \arg\max_{(\mathbf{p}_j,\mathbf{w}_j)\geq 0} \sum_{i=1}^{N} l_i \min\{r_{j,i}^{[k-1]}, C_{j,i}^*(p_{j,i})\} \tag{6a}$$

$$\text{s.t.} \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n} E_{j,i}, \tag{6b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq E_{j,max}, \tag{6c}$$

$$\forall n = 1, 2, \ldots, N, \tag{6d}$$

for $j = 1, 2$, and

$$(\mathbf{r}_j^{[k]}, \mathbf{d}_j^{[k]}) = \arg\max_{(\mathbf{r}_j,\mathbf{d}_j)\geq 0} \sum_{i=1}^{N} l_i \min\{r_{j,i}, C_{j,i}^*(p_{j,i}^{[k]})\} \tag{7a}$$

$$\text{s.t.} \sum_{i=1}^{n}(l_i r_{j,i}+d_{j,i}) \leq \sum_{i=1}^{n} B_{j,i}, \tag{7b}$$

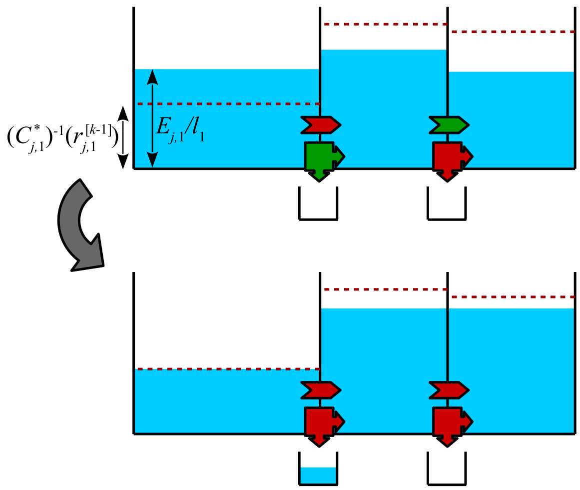

Fig. 2. Directional waterfilling with right permeable taps and pumps for the energy problem for a set up with three epochs. Initial allocations are shown at the top, and the optimal allocations are shown at the bottom.

$$\sum_{i=1}^{n} B_{j,i} - \sum_{i=1}^{n}(l_i r_{j,i} + d_{j,i}) \leq B_{j,max}, \quad (7c)$$

$$\forall n = 1, 2, \ldots, N. \quad (7d)$$

In order to solve (6), i.e., the energy problem, we derive the stationarity condition on p_n for all $n = 1, 2, \ldots, N$. The optimal solution of (6) admits a directional waterfilling interpretation as follows. Whenever $C^*_{j,n}(p_{j,n}) < r^{[k-1]}_{j,n}$, there is sufficient data allocated by (7) in the previous iteration; thus, the solution is the same as the directional waterfilling solution of [3] for an infinite backlog of data and an infinite data buffer. In other words, energy can flow only from past epochs to future epochs via right permeable taps between the water bins that signify the epochs. A tap turns off when the total amount of energy (water) that has flown through it reaches $E_{j,max}$ for node T_j, $j = 1, 2$.

If there is not enough data to fully utilize the initial amount of energy available in an epoch, then we must have $C^*_{j,n}(p_{j,n}) = r^{[k-1]}_{j,n}$. The optimality conditions imply that $p_{j,n}$ is decreased until this condition is satisfied. That is, no more than $(C^*_{j,n})^{-1}(r^{[k-1]}_{j,n})$ units of power are allowed in each epoch. This means that the data allocated by (7) in the previous iteration results in a maximum power of $(C^*_{j,n})^{-1}(r^{[k-1]}_{j,n})$ for epoch n. We interpret this phenomenon by introducing water pumps and overflow protection bins to the waterfilling solution. The water pump for the nth epoch is inactive as long as $p_{j,n} \leq (C^*_{j,n})^{-1}(r^{[k-1]}_{j,n})$. However, if this constraint is violated by the initial water levels, or the operation of the right permeable taps, then the water pump is activated. The water pump is responsible for bringing the water level down to $(C^*_{j,n})^{-1}(r^{[k-1]}_{j,n})$. In order to do this, it pumps water to the next bin. When the right permeable tap turns off since the battery is full in the next epoch, it pumps water into the overflow protection bin. Here, the transfer of water to an overflow protection bin models a positive w_n, i.e., wasted energy. Note that this excess amount of energy is unable to improve the objective due to the limitation on the available data, thus the transmitter can afford to waste it.

Algorithm 1 The proposed epoch-by-epoch solution for the data problem for node T_j, $j = 1, 2$.

1: Initialize $n = 1$.
2: Optimize epoch n using
   $r_{j,n} = \min\{B_{j,n}/l_n, C^*_{j,n}(p^{[k]}_{j,n})\}$.
3: Determine how much data to drop using
   $d_{j,n} = (B_{j,n} - l_n C^*_{j,n}(p^{[k]}_{j,n}) - B_{j,max})^+$.
4: Update the next data arrival using
   $B_{j,n+1} := B_{j,n+1} + \min\{B_{j,n} - l_n C^*_{j,n}(p^{[k]}_{j,n}), B_{j,max}\}$.
5: **if** $n < N$ **then**
6:   $n := n + 1$.
7:   **go to** 2.
8: **else**
9:   **return** $(\mathbf{r}_j, \mathbf{d}_j)$.
10: **end if**

An example is shown in Fig. 2 with three epochs. There is not enough data in the first epoch, thus some water is pumped to the second epoch by the water pump even though the water level, i.e., the average power, is higher in the second epoch. When the pump can no longer pump any water to the second epoch, the remaining water is transferred into the overflow protection bin.

For (7), i.e., the data problem, we propose an algorithmic solution. The solution finds the optimal $r_{j,n}$ and $d_{j,n}$ values on an epoch-by-epoch basis, starting with the first epoch, and updating the next data arrival. With the updated data arrival, the next epoch can be treated as the first arrival for a subproblem that starts with the second epoch. The solution is given in Algorithm 1 where $(x)^+ = \max\{x, 0\}$. The proof of optimality is by induction, i.e., assuming optimality for the past epochs, the algorithm optimizes the current epoch and moves on to the next one.

With the optimal solutions to the energy and data problems identified, the description of the optimal transmission policy for a two-way channel is complete.

III. Throughput Maximization for the Two-Way Relay Channel

A. System Model

Consider now an AWGN half duplex two-way relay channel with energy harvesting transmitters T_1 and T_2 and an energy harvesting relay T_0 as in Fig. 3. The AWGN at any receiving node is zero mean and unit variance. The power gain from T_i to T_j is h_{ij} with $h_{12} = h_{21} = 0$, i.e., there is no direct link between T_1 and T_2. Node T_i employs a finite battery of capacity $E_{i,max}$, and transmitter T_i employs a finite data buffer of capacity $B_{i,max}$. In the nth epoch, node T_i harvests $E_{i,n}$ ($\leq E_{i,max}$) units of energy, and transmitter T_i receives $B_{i,n}$ ($\leq B_{i,max}$) units of data at a constant rate. The relaying strategy is chosen to be decode-and-forward for clarity of exposition.

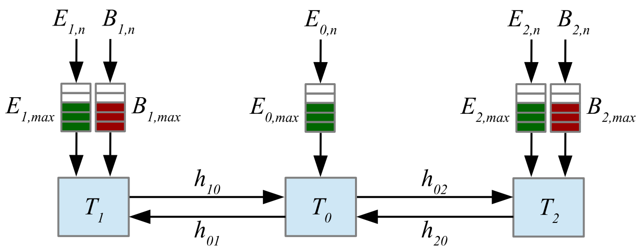

Fig. 3. The energy harvesting two-way relay channel with a finite battery at each node and a finite data buffer at each transmitter.

B. Problem Statement and Solution

The throughput maximization problem for this set up is

$$\max_{\substack{\mathbf{p},\mathbf{w},\mathbf{r},\mathbf{d}\geq 0,\\ 0\leq \boldsymbol{\Delta}\leq 1}} \sum_{i=1}^{N} l_i(r_{1,i}+r_{2,i}) \tag{8a}$$

$$\text{s.t. } \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n} E_{j,i}, \tag{8b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq E_{j,max}, \tag{8c}$$

$$\sum_{i=1}^{n}(l_i r_{k,i}+d_{k,i}) \leq \sum_{i=1}^{n} B_{k,i}, \tag{8d}$$

$$\sum_{i=1}^{n} B_{k,i} - \sum_{i=1}^{n}(l_i r_{k,i}+d_{k,i}) \leq B_{k,max}, \tag{8e}$$

$$(r_{1,n}, r_{2,n}) \in \mathcal{R}(p_{0,n}, p_{1,n}, p_{2,n}, \Delta_n), \tag{8f}$$

$$\forall j=0,1,2,\ k=1,2,\ n=1,2,\dots,N \tag{8g}$$

where

$$\begin{aligned}\mathcal{R}(p_{0,n}, p_{1,n}, p_{2,n}, \Delta_n) = \{(r_1, r_2):&\\ 0 \leq r_1 \leq \min\{\Delta_n C(\frac{h_{10}p_{1,n}}{\Delta_n}), (1-\Delta_n)C(\frac{h_{02}p_{0,n}}{1-\Delta_n})\},&\\ 0 \leq r_2 \leq \min\{\Delta_n C(\frac{h_{20}p_{2,n}}{\Delta_n}), (1-\Delta_n)C(\frac{h_{01}p_{0,n}}{1-\Delta_n})\},&\\ r_1 + r_2 \leq \Delta_n C(\frac{h_{10}p_{1,n}+h_{20}p_{2,n}}{\Delta_n})\}&\end{aligned} \tag{9}$$

denotes the set of achievable rates given the transmit powers at all three nodes [14]. Here, Δ_n is the fraction of the nth epoch scheduled for the multiple access phase where the two transmitters transmit their messages to the relay.

In order to solve (8), we follow the same approach as we did for the two-way channel, and eliminate Δ_n as a decision variable. The Karush-Kuhn-Tucker analysis of (8) yields that there exists at least one optimal solution to (8) where $r_{1,n} + r_{2,n} = \Delta_n C((h_{10}p_{1,n}+h_{20}p_{2,n})/\Delta_n)$ for all $n = 1, 2, \dots, N$. Hence, we define the best achievable sum rate given transmit powers $f(p_{0,n}, p_{1,n}, p_{2,n})$ equal to

$$\max_{\substack{r_{1,n}, r_{2,n},\\ 0\leq\Delta_n\leq 1}} r_{1,n}+r_{2,n} \tag{10a}$$

$$\text{s.t. } (r_{1,n}, r_{2,n}) \in \mathcal{R}(p_{0,n}, p_{1,n}, p_{2,n}, \Delta_n), \tag{10b}$$

$$r_{1,n}+r_{2,n} = \Delta_n C((h_{10}p_{1,n}+h_{20}p_{2,n})/\Delta_n). \tag{10c}$$

We omit the details of the optimal solution of (10) due to space limitations. The resulting best achievable sum rate is

$$f(p_{0,n}, p_{1,n}, p_{2,n}) = \min_{\Delta\in\mathcal{D}} \Delta C(\frac{h_{10}p_{1,n}+h_{20}p_{2,n}}{\Delta}) \tag{11}$$

where $\mathcal{D} = \{\Delta_1^*, \Delta_2^*, \Delta_3^*\}$ is found by solving

$$\begin{aligned}&\Delta_1^* C((h_{10}p_{1,n}+h_{20}p_{2,n})/\Delta_1^*)\\ &\qquad= (1-\Delta_1^*)[C(\frac{h_{01}p_{0,n}}{1-\Delta_1^*}) + C(\frac{h_{02}p_{0,n}}{1-\Delta_1^*})],\end{aligned} \tag{12a}$$

$$\Delta_2^* C(\frac{h_{20}p_{2,n}}{h_{10}p_{1,n}+\Delta_2^*}) = (1-\Delta_2^*)C(\frac{h_{02}p_{0,n}}{1-\Delta_2^*}), \tag{12b}$$

$$\Delta_3^* C(\frac{h_{10}p_{1,n}}{h_{20}p_{2,n}+\Delta_3^*}) = (1-\Delta_3^*)C(\frac{h_{01}p_{0,n}}{1-\Delta_3^*}). \tag{12c}$$

Equations (12) admit unique solutions that can be found numerically. As a result, (8) becomes

$$\max_{\mathbf{p},\mathbf{w},\mathbf{r},\mathbf{d}\geq 0} \sum_{i=1}^{N} l_i(r_{1,i}+r_{2,i}) \tag{13a}$$

$$\text{s.t. } \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq \sum_{i=1}^{n} E_{j,i}, \tag{13b}$$

$$\sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n}(l_i p_{j,i}+w_{j,i}) \leq E_{j,max}, \tag{13c}$$

$$\sum_{i=1}^{n}(l_i r_{k,i}+d_{k,i}) \leq \sum_{i=1}^{n} B_{k,i}, \tag{13d}$$

$$\sum_{i=1}^{n} B_{k,i} - \sum_{i=1}^{n}(l_i r_{k,i}+d_{k,i}) \leq B_{k,max}, \tag{13e}$$

$$r_{1,n}+r_{2,n} \leq f(p_{0,n}, p_{1,n}, p_{2,n}), \tag{13f}$$

$$\forall j=0,1,2,\ k=1,2,\ n=1,2,\dots,N. \tag{13g}$$

We can now replace the objective of (13) with

$$\sum_{i=1}^{N} l_i \min\{r_{1,i}+r_{2,i}, f(p_{0,i}, p_{1,i}, p_{2,i})\} \tag{14}$$

and remove constraint (13f). Now, the feasible region of (13) is again separable between energy variables $(\mathbf{p}, \mathbf{w})$ and data variables $(\mathbf{r}, \mathbf{d})$. Moreover, we can separate (13) between the three nodes. Consequently, we obtain energy problems for all nodes, and data problems for nodes T_1 and T_2. The solutions of the resulting problems are the same as the waterfilling and induction based solutions given in Section II-B.

IV. Numerical Results

We consider the two-way relay channel in our simulations to investigate the interaction between the sizes of the two data buffers at nodes T_1 and T_2, and their collective impact on the achieved throughput. The results are shown in Fig. 4. The peak energy harvest rates and the battery sizes are set at 50 mJ for all nodes. The noise density at all nodes is 10^{-19} W/Hz, and the power gain in all links is -110 dB. A bandwidth of 1 MHz is available for two-way communication, which takes

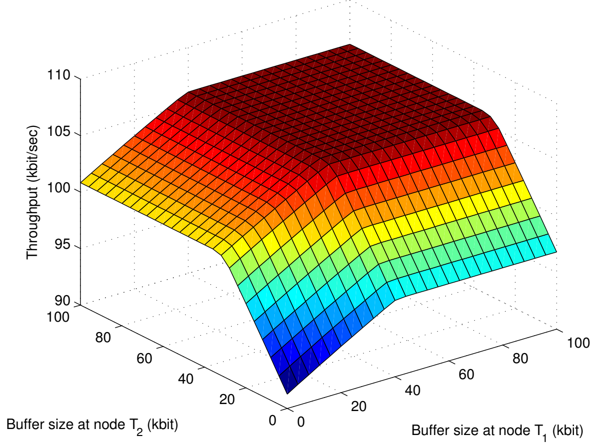

Fig. 4. Throughput values achieved by the optimal transmission policy versus varying buffer sizes at nodes T_1 and T_2.

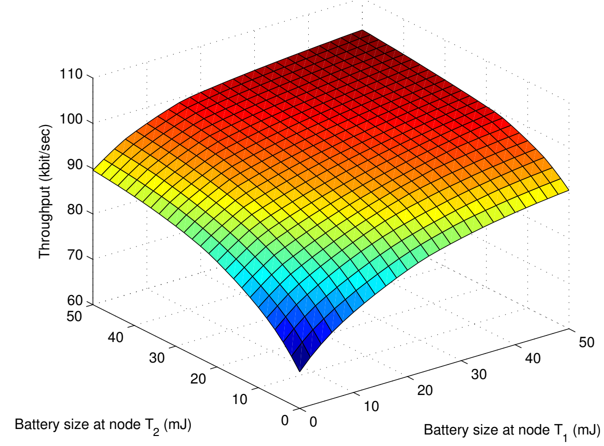

Fig. 5. Throughput values achieved by the optimal transmission policy versus varying battery sizes at nodes T_1 and T_2.

place for 10 seconds. The buffer sizes are varied from 0 kbits (no buffer) to 100 kbits for nodes T_1 and T_2. We observe that the optimal throughput is saturated at rather modest values of the buffer sizes for both nodes, yielding values as if the buffers are unlimited. On average, 0.4 seconds of incoming data need to be stored in the data buffers for optimal throughput. Note that communication is possible without any data buffers since the transmitters can send the data they receive in an epoch within the same epoch. However, they are unable to save the data for more opportune times. The existence of data buffers thus enables throughput improvement.

Fig. 5 demonstrates the impact of the battery sizes at nodes T_1 and T_2 on the achieved throughput. The simulation set up is the same as the previous one, except the buffer sizes for nodes T_1 and T_2 are set at 50 kbits. The battery sizes for nodes T_1 and T_2 are varied from 0 mJ to 50 mJ. Similarly to the previous result with varied buffer sizes, Fig. 5 offers insights into choosing the battery sizes for optimal throughput which saturates beyond certain battery sizes, here 50 mJ.

V. Conclusion

In this paper, we have studied throughput maximization for the two-way channel and the two-way relay channel with energy harvesting nodes. We have considered the realistic case of finite capacity batteries as well as finite capacity data buffers. We have shown that the throughput maximization problems in either channel model can be decomposed into an energy problem and a data problem, and subsequently can be solved using alternating maximization. We have shown that the energy problem admits a directional waterfilling solution [3] with the addition of the new notions of water pumps and overflow protection bins. We have solved the data problem using forward induction where we identify optimal data amounts to transmit on an epoch-by-epoch basis. We have provided numerical results to verify our analytical findings. In particular, we have observed that the optimal throughput saturates as the buffer and battery sizes increase, and the required buffer size is only a fraction of the amount of total data the optimal throughput renders.

References

[1] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Transactions on Communications*, vol. 60, no. 1, pp. 220–230, 2012.

[2] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1180–1189, 2012.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1732 –1743, Sep. 2011.

[4] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *Journal of Communications and Networks*, vol. 14, no. 2, pp. 151 –161, 2012.

[5] D. Gunduz and B. Devillers, "Two-hop communication with energy harvesting," in *Proceedings of the 4th IEEE CAMSAP*, Dec. 2011.

[6] O. Orhan and E. Erkip, "Throughput maximization for energy harvesting two-hop networks," in *Proceedings of the IEEE International Symposium on Information Theory, ISIT*, Jul. 2013.

[7] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *Journal of Communications and Networks*, vol. 14, no. 2, pp. 140–150, 2012.

[8] K. Tutuncuoglu, B. Varan, and A. Yener, "Energy harvesting two-way half-duplex relay channel with decode-and-forward relaying: Optimum power policies," in *Proceedings of the 18th International Conference on Digital Signal Processing, DSP*, Jul. 2013.

[9] B. Varan and A. Yener, "Multi-pair and multi-way communications using energy harvesting nodes," in *Proceedings of the 47th Asilomar Conference on Signals, Systems and Computers*, Nov. 2013.

[10] K. Tutuncuoglu and A. Yener, "Communicating with energy harvesting transmitters and receivers," in *Proceedings of the 2012 Information Theory and Applications Workshop*, Feb. 2012.

[11] M. Gregori and M. Payaro, "Energy-efficient transmission for wireless energy harvesting nodes," *IEEE Transactions on Wireless Communications*, vol. 12, no. 3, pp. 1244–1254, March 2013.

[12] B. Varan and A. Yener, "Energy harvesting communications with energy and data storage limitations," in *Proceedings of the 2014 IEEE Global Communications Conference Communication Theory Symposium, GLOBECOM'14*, Dec. 2014.

[13] D. P. Bertsekas, *Nonlinear programming*. Belmont, MA: Athena Scientific, 1999.

[14] S. Kim, N. Devroye, P. Mitran, and V. Tarokh, "Achievable rate regions and performance comparison of half duplex bi-directional relaying protocols," *IEEE Transactions on Information Theory*, vol. 57, no. 10, pp. 6405–6418, 2011.