Multi-pair and Multi-way Communications Using
Energy Harvesting Nodes

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Abstract—In this paper, we consider multi-directional information flow in networks with energy harvesting nodes and intermittent data arrivals at the source nodes. The sum throughput optimization problem is studied in models where an intermediate energy harvesting relay facilitates the multi-directional information exchange in the network. In particular, we consider the multi-way relay channel as well as its special case with two-user clusters, i.e., the multi-pair bidirectional relay channel, with a decode-and-forward relay. A new ingredient that is introduced in this study is the finite size data buffers at each node in addition to the finite size energy storage units (batteries). We observe that the optimal transmission policies can be found by solving the underlying convex optimization problem. We present numerical results for the multi-way and multi-pair setups to demonstrate the throughput performance of the optimal policies and the impact of the finite buffers.

I. INTRODUCTION

Wireless communication using energy harvesting nodes has recently emerged as a viable green solution to prolonging network lifetime. While energy efficient operation is useful and desirable for all wireless networks, the introduction of wireless transmitters with rechargeable batteries, that harvest energy from external (natural or man-made) sources, makes carefully designed transmission policies amortizing the available energy the key design element. Such transmission policies for the single-user channel is introduced in [1] where the transmission completion time minimization problem is solved. References [2], [3] have solved the single-user throughput maximization problem with the addition of finite energy storage, i.e., finite-capacity batteries for static and fading channels respectively. Several multi-user channels have also been studied, see for example [4]–[9] and many others. Of particular relevance to this work are models where a relay node is the means to facilitate communication, these include recent references [9] where a two-hop relay channel is considered with infinite buffer and battery sizes, and [10] which identifies a special case of this model where the data buffer at the relay is immaterial. Additionally, the two-way relay channel is studied in recent references [11], [12] where the optimal transmission policies are found for a decode-and-forward relay and a relay that can adaptively switch between relaying strategies for further throughput improvement.

In this paper, we consider the multi-way relay channel which generalizes communication models where information in the network flows solely through the intermediate shared relay [13]. In particular, the $L$ cluster multi-way relay channel models $K$ wireless nodes in each cluster each of which wishes to exchange messages with the remaining nodes in the cluster, but can do so only using the relay node shared by all the clusters. The model’s special cases include the two-hop relay channel, the two-way relay channel and the multi-pair bidirectional relay channel (with $K = 2$ users in each cluster).

We consider the case where all of the nodes in the network are energy harvesting with finite capacity batteries. Furthermore, different from previous works on identifying optimum transmission policies for energy harvesting networks which assumed unlimited data buffers at the nodes, we introduce limited data buffers at each node. The goal of this addition to the model is to consider a more realistic energy harvesting wireless communication scenario where both data and energy are intermittently available during the communication session and need to be stored in limited capacity data and energy buffers. The resulting setup allows for adopting the data storage at the nodes in order to maximize throughput.

We formulate and solve the sum throughput maximization problem for the multi-way relay channel. We employ a decode-and-forward relay as it is shown in [13] that decode-and-forward outperforms other relaying strategies at low transmit power, a condition that we expect in an energy harvesting network, and with a relatively large number of users, a condition that we would like to have. We show that the throughput maximization problem can be expressed as a convex optimization problem which we then solve using a steepest descent algorithm. We present numerical results demonstrating the throughput performance of the optimal policies as compared to naïve transmission policies, as well as the impact of the buffer size at the nodes on the resulting sum throughput.

II. SYSTEM MODEL

Consider a Gaussian decode-and-forward multi-way relay channel with $L \geq 1$ clusters of users where each cluster has $K \geq 2$ half-duplex and single-antenna nodes and one half-duplex relay node as shown in Fig. 1. We denote the relay node with $T_0$, and the $i$th node in the $j$th cluster with $T_{j,i}$ for $i \in I_K \triangleq \{1, 2, \ldots, K\}$ and $j \in I_L \triangleq \{1, 2, \ldots, L\}$. Each source node is interested exchanging messages with all the other source nodes in its cluster. The source nodes do not
have direct links and can only communicate through the shared relay. Communication takes place in two phases. Phase I refers to the uplink phase, i.e., the multiple-access phase where the relay listens to the source nodes and decodes all the messages. Phase II refers to the downlink phase, i.e., the broadcast phase where the relay transmits functions of the decoded messages to the $LK$ source nodes. In phase I, the power gain from $T_{j,i}$ to $T_1$ is denoted by $h_{j,i}^I$, and in phase II, the power gain from $T_0$ to $T_{j,i}$ is denoted by $h_{j,i}^I$. We normalize these power gains so that the noise at each node is zero-mean and unit-variance.

Each node harvests amounts of energy during the communication session, and the source nodes $T_{j,i}$ also receive data packets during the communication session. We refer to the time duration between any two consecutive data packets or data arrivals at any nodes as an *epoch*. $N$ denotes the number of epochs by the deadline, $T$. $s_n$ denotes the beginning of the $n$th epoch and we set $s_{N+1} = T$. The length of the $n$th epoch is denoted by $t_n = s_{n+1} - s_n$, $n \in I_N = \{1, 2, \ldots, N\}$. Node $T_{j,i}$ harvests $E_{j,i}^{(n)}$ units of energy and $B_{j,i}^{(n)}$ bits of data at $s_n$, for $i \in I_K$ and $j \in I_L$. The relay node $T_0$ harvests $E_0^{(n)}$ units of energy at $s_n$. If a node does not harvest any energy or any data at $s_n$ for some $n$, then we simply set the corresponding energy or data arrival amount to zero. Each node employs a finite-capacity battery which, at source node $T_{j,i}$, can at most store $E_{j,i}^{(n)}$ units of energy and at the relay node $T_0$, can store up to $E_0^{(n)}$ units of energy. For the data arrivals, each source node has a finite-capacity buffer that it can use to store data packets until the packets can be transmitted. The data buffer at source node $T_{j,i}$ can store at most $B_{j,i}^{(n)}$ bits of data, $i \in I_K$, $j \in I_L$. Similarly, the relay node employs $LK$ finite-capacity buffers to store incoming data from the source nodes, specifically buffer $(j,i)$ with finite capacity $B_{j,i}^{(n)}$ is used to store the messages of node $T_{j,i}$, $i \in I_K$, $j \in I_L$.

All arrival instants $s_n$, energy amounts $E_{j,i}^{(n)}$, $E_0^{(n)}$ and data amounts $B_{j,i}^{(n)}$ are assumed to be known non-causally as in references [1]–[5], [9]–[12]. $\Delta_0^{(n)} \in [0, 1]$ denotes the fraction of the $n$th epoch during which the sources transmit and the relay listens (phase I). $1 - \Delta_0^{(n)}$ denotes the fraction of the $n$th epoch during which the relay forwards the messages it has received (phase II). Phase I is simply a multiple-access channel with $LK$ transmitters, $T_{j,i}$, and a receiver, $T_0$ leading to the rate region [13] as

$$\sum_{j \in S_L} \sum_{i \in S_K} R_{j,i} \leq C \left( \sum_{j \in S_L} \sum_{i \in S_K} h_{j,i}^I P_{j,i} \right),$$

(1)

for all $S_L \subseteq I_L$ and $S_K \subseteq I_K$ where $C(x) = \frac{1}{2} \log(1 + x)$. $R_{j,i}$ denotes the rate achieved from $T_{j,i}$ to $T_0$, and $P_{j,i}$ denotes the transmit power at node $T_{j,i}$, $i \in I_K$, $j \in I_L$. Phase II amounts to a broadcast channel with side information. We adopt the broadcast phase of the decode-and-forward scheme given in [13] with time-sharing among clusters. Phase II in any epoch is divided into $L$ time slots proportional to $\tau_l \geq 0$ where $\sum_{l=1}^{L} \tau_l = 1$. The relay power $P_0$ is also divided into $L$ fractions, $P_{0,j}$, $j \in I_L$. In the $j$th time slot, the relay node broadcasts the messages of all users in cluster $j$ to all users in the cluster with transmit power $P_{0,j}$. The messages can be decoded by all the users if the rates $R_{j,i}$ satisfy

$$\sum_{i \in I_K} R_{j,i} \leq \tau_j C \left( h_{j,i}^I P_{0,j} \right),$$

(2)

for all $j \in I_L$ and $l \in I_K$. We denote the fraction of the time slot in phase II of the $n$th epoch in which the relay node broadcasts to cluster $j$ by $\Delta_0^{(n)}$. Thus, we have $\Delta_0^{(n)} = \tau_j(1 - \Delta_0^{(n)})$ and $\sum_{j=1}^{L} \Delta_0^{(n)} = 1$. The length of the time slot reserved for broadcast to cluster $j$ in epoch $n$ is then given by $\Delta_0^{(n)} t_n = \tau_j(1 - \Delta_0^{(n)}) t_n$.

Since the achievable rates in (1) and (2) are concave in transmit powers, by following the steps in [1, Lemma 2], we can conclude that the transmit power at each node should remain constant throughout an epoch while the node is transmitting. We denote the average transmit power at source node $T_{j,i}$ in the $n$th epoch by $P_{j,i}^{(n)}$, and the average transmit power at the relay node $T_0$ for cluster $j$ in the $n$th epoch by $P_{0,j}^{(n)}$. The transmit powers are averaged over the duration of the corresponding epoch, i.e., node $T_{j,i}$ transmits with power $p_{j,i}^{(n)} / \Delta_0^{(n)}$ for a duration of $\Delta_0^{(n)} t_n$ seconds. Similarly, we denote by $r_{j,i}^{(n)}$ the average rate achieved from $T_{j,i}$ to $T_0$ in phase I, and by $r_{j,i}^{(n)}$ the average rate of node $T_{j,i}$’s message that the relay forwards in phase II of the $n$th epoch.

### III. SUM THROUGHPUT MAXIMIZATION

In this section, we express the throughput maximization problem for the multi-way relay channel and solve it. We
start with the feasibility conditions that have to be satisfied by the optimization parameters, \( p^{(n)}_{j,i}, p^{(n)}_{0,j}, r^{I,n}_{j,i}, r^{II,n}_{j,i}, \Delta^{(n)}_j \), and \( \Delta_j^{(n)} \), for \( j \in I_L, i \in I_K, n \in I_N \). Since all nodes are energy harvesting, feasible policies must schedule power values that satisfy the energy causality constraint, i.e., the nodes cannot spend energy before they harvest it, which can be expressed as

\[
\sum_{n=1}^{\bar{n}} l_n p^{(n)}_{j,i} \leq \sum_{n=1}^{\bar{n}} E^{(n)}_{j,i},
\]

(3a)

\[
\sum_{n=1}^{\bar{n}} L \sum_{j=1}^{\bar{n}} l_n p^{(n)}_{0,j} \leq \sum_{n=1}^{\bar{n}} E_0^{(n)},
\]

(3b)

where \( \bar{n} \) denotes the index of the epoch, at the beginning of which (3) has to be satisfied. Additionally, each node has a finite-capacity battery, i.e., it can store as much energy as its battery allows. Therefore, we get the following battery constraint for the source nodes and the relay:

\[
\sum_{n=1}^{\bar{n}} E^{(n)}_{j,i} - \sum_{n=1}^{\bar{n}-1} l_n p^{(n)}_{j,i} \leq E^{(max)}_{j,i},
\]

(4a)

\[
\sum_{n=1}^{\bar{n}} E_0^{(n)} - \sum_{n=1}^{\bar{n}-1} \sum_{j=1}^{L} l_n p^{(n)}_{0,j} \leq E^{(max)}_0.
\]

(4b)

Additionally, a source node cannot transmit the data it has not yet received, and the amount of data it can store is limited by its buffer size, leading to the following constraints

\[
\sum_{n=1}^{\bar{n}} l_n I^{I,n}_{j,i} \leq \sum_{n=1}^{\bar{n}} B^{(n)}_{j,i},
\]

(5)

\[
\sum_{n=1}^{\bar{n}} B^{(n)}_{j,i} - \sum_{n=1}^{\bar{n}-1} l_n I^{I,n}_{j,i} \leq B^{(max)}_{j,i}.
\]

(6)

The achievable rates in each epoch have to be in the rate regions in (1) and (2) as

\[
\sum_{j \in S_L, i \in S_K} r^{I,n}_{j,i} \leq 0 - \Delta^{(n)}_0 C \frac{\sum_{j \in S_L} \sum_{i \in S_K} h^{I,n}_{j,i} p^{(n)}_{j,i}}{\Delta_0^{(n)}},
\]

in phase I for all \( S_L \subset I_L \) and \( S_K \subset I_K \), and

\[
\sum_{i \in S_K, j \notin S_L} r^{II,n}_{j,i} \leq \Delta^{(n)}_j C \frac{h^{II,n}_{j,i} p^{(n)}_{0,j}}{\Delta_j^{(n)}},
\]

in phase II for all \( j \in I_L \) and \( l \in I_K \). In addition, all optimization parameters must be non-negative and the fractions of time slots in each epoch, \( \Delta^{(n)}_j \), \( j=0,1,\ldots L \), have to sum up to 1. Finally, with all the constraints given above, the sum throughput optimization problem becomes

\[
\max \sum_{n=1}^{N} \sum_{j=1}^{L} \sum_{i=1}^{K} l_n r^{I,n}_{j,i} p^{(n)}_{j,i} + r^{II,n}_{j,i} p^{(n)}_{0,j}, \Delta^{(n)}_0, \Delta^{(n)}_j
\]

s.t. \( 0 \leq \Delta^{(n)}_0, \Delta^{(n)}_j \leq 1, \sum_{j=0}^{L} \Delta^{(n)}_j = 1, \) \( p^{(n)}_{j,i}, p^{(n)}_{0,j}, r^{I,n}_{j,i}, r^{II,n}_{j,i} \geq 0, \) and (3)–(9), \( \forall n, i \in I_N, j \in I_L, l \in I_K, S_L \subset I_L, S_K \subset I_K \).

(10)

The constraints in (3)–(7) and the objective function in (10) are linear. The constraints in (8) and (9) do not violate convexity of the feasible region since the right hand sides in (8) and (9) are of the form \( yC(x/y) \) for some \( x \) and \( y \), which is jointly concave in \( x \) and \( y \) since it is the perspective of a concave function, \( C(x) \) [14, §3.2.6]. Thus, (10) is a convex program which can be solved by a number of iterative algorithms. We choose the method of steepest descent which is guaranteed to converge to an optimal solution. The algorithm is provided in Algorithm 1 where \( x = (p^{(n)}_{j,i}, p^{(n)}_{0,j}, r^{I,n}_{j,i}, r^{II,n}_{j,i}, \Delta^{(n)}_0, \Delta^{(n)}_j) \) is a solution, \( x^{(k)} \) is the solution at the \( k \)th iteration, \( f(x) = \sum_{n=1}^{N} \sum_{j=1}^{L} \sum_{i=1}^{K} l_n r^{I,n}_{j,i} p^{(n)}_{j,i} + r^{II,n}_{j,i} p^{(n)}_{0,j} \Delta^{(n)}_0, \Delta^{(n)}_j \) is the objective to be maximized, and \( \Pi(\cdot) \) is a projection operator which projects its argument onto the feasible space in (10).

It is worthwhile to note that one needs to leave the rates \( r^{I,n}_{j,i} \) and \( r^{II,n}_{j,i} \) as optimization parameters in (10) instead of expressing them in terms of \( p^{(n)}_{j,i}, p^{(n)}_{0,j}, \Delta^{(n)}_0, \Delta^{(n)}_j \). This is necessary because the feasible space as given in (10) is not strictly concave.

Algorithm 1 Iterative algorithm for sum throughput optimization.

1: Pick precision \( \epsilon > 0 \), resolution \( \rho \in \{1, 2, \ldots \} \), and maximum step size \( \delta_{(\text{max})} > 0 \).
2: Initialize \( x^{(0)} \).
3: \( k = 0 \)
4: repeat
5: \( k = k + 1 \)
6: \( v^* = 0 \)
7: \( \delta^* = 0 \)
8: for \( i = 0 \) to \( \rho \) do
9: \( \delta = (i/\rho)\delta_{(\text{max})} \)
10: \( v = f(\Pi(x^{(k-1)} + \delta \nabla_x f(x^{(k-1)}))) \)
11: if \( v > v^* \) then
12: \( v^* = v \)
13: \( \delta^* = \delta \)
14: end if
15: end for
16: \( x^{(k)} = \Pi(x^{(k-1)} + \delta^* \nabla_x f(x^{(k-1)})) \)
17: until \( \|x^{(k)} - x^{(k-1)}\| < \epsilon \)
18: return \( x^{(k)} \)

1Observe that (10) may have multiple optima resulting in the same sum throughput as the objective is not strictly concave.
is unlike previous work on energy harvesting, e.g., [1], [2], [4], [11] and is due to the interactive nature of the channel model: the nodes may have to operate at a lower rate and use less power than the power dictated by the average power constraint. As an example, consider a setup with a the relay that has data only from node $T_{1,1}$ in the last epoch. Then, $r_{j,i}^{F,n}$ has to be zero for $j \in I_L \setminus \{1\}, i \in I_K$ since the relay does not have any data from clusters $2, 3, \ldots, L$ to forward. However, some energy still needs to be spent to forward node $T_{1,1}$’s message to cluster 1. Thus, the relay uses its energy to forward only one message to cluster 1, although it would have been possible to forward messages from $T_{1,i}, i \in I_K \setminus \{1\}$ to cluster 1 if there were any data to forward.

As a final remark, we note that the multi-way relay channel model has interesting special cases that can also provide us with insights into optimality with energy harvesting. These special cases include the multi-pair relay channel, the two-way relay channel, the two-hop channel, the two-way channel, and the single-user channel for which the throughput-maximizing transmission policy can be found by solving (10) with the appropriate substitutions. The multi-pair relay channel consists of $L$ pairs of source nodes and a relay where each source node is only interested in the message of the other user in its pair, i.e., a multi-way relay channel with $K = 2$. The sum throughput optimal policy for the energy harvesting multi-pair relay channel thus can be found by solving (10) with $K = 2$, which will be demonstrated in the next section with a numerical example.

### IV. Numerical Results

We present an optimal policy for the multi-way relay channel in Fig. 2(a). The feasible energy tunnel is shown in the figure where the upper wall represents the cumulative harvested energy and the lower wall represents the energy storage, i.e., batter size constraint. As is evident from the figure, the optimal policy is not necessarily the shortest path in the tunnel, which was the case for [1] and [2]. The observation that $\Delta_0^{(1)}$ is low suggests that the relay uses its buffers to store data to transmit in the last epoch. Fig. 2(b) shows the sum throughput maximizing policy in a multi-pair relay channel. Similarly, we see that the optimal policy does not have to be the shortest path in the feasible energy tunnel. We also observe that $\Delta_0^{(1)}$ is high. This is because node $T_{3,2}$ harvests a very low amount of energy at the beginning of transmission, and the buffers at the relay are empty at the beginning of transmission. Thus, in order to utilize the relay power efficiently, more time has to be spared for the source nodes to transmit their data to the relay.

Fig. 3(a) shows the sum throughput of the optimal policy, along with lower- and upper-bounds for comparison in a multi-way relay channel setup with $L = 3$ clusters that each contain $K = 3$ users, unit channel gains, $E_0^{(\text{max})} = 5 \text{ J}$, $E_j^{(\text{max})} = 5 \text{ J}$, $B_j^{(\text{max})} = 10 \text{ kbit}$, and $B_j^{(\text{max})} = 10 \text{ kbit}$, for $j = 1, 2, 3, i = 1, 2, 3$. The peak energy harvesting rate is fixed at $5 \text{ J}$ for all nodes except $T_{1,1}$, and varied from $0 \text{ J}$ to $5 \text{ J}$ for $T_{1,1}$.

2The energy tunnel refers to the space of policies that satisfy the energy causality constraint (3) and the battery constraint (4).
The upper-bound corresponds to the case where all nodes are assumed to receive all the energy at the beginning of transmission with infinite batteries. This assumption removes constraints (3) and (4), and results in a larger feasible region for (10). The lower-bound is the hasty policy where the nodes do not have batteries, so they cannot store energy for later epochs, and also the relay does not have data buffers. As can be seen in Fig. 3(a), the throughput curve achieved by the optimal policy is monotonically increasing and concave in the peak harvest rate for node $T_{1,1}$. It is significantly higher than the lower-bound achieved by the hasty policy, which demonstrates how batteries and data buffers can help achieve higher performance in an energy harvesting network. The throughput values achieved by the optimal policy are very close to the upper-bound under energy deficient conditions which are more likely to occur in an energy harvesting setup.

Lastly, Fig. 3(b) and 3(c) show how the buffer size affects the achievable throughput in the multi-way relay channel setup given above. The sizes of the buffers are set to be equal, and this value is varied. As can be seen, larger buffers allow the relay to store more data for later epochs when the relay can more efficiently forward messages, and naturally, the achieved throughput increases in buffer size. However, after a certain buffer size, we observe that the achieved throughput saturates leading to the design insight that we can choose relatively modest size buffers for throughput optimal operation.

V. CONCLUSION

In this paper, we have studied a multi-way relay channel consisting of energy harvesting nodes with finite batteries, finite data buffers, and intermittent data arrivals. We have seen that the sum throughput maximization problem in this set up is convex and solved it using the method of steepest descent. We have observed that optimal policy requires careful coordination of the transmission, energy and data storage policies of the users and the relay. Additionally, we have observed that the data buffer sizes do impact the achievable sum throughput, although a modest buffer size appears to be sufficient to achieve the performance that would result with unlimited buffers. Future work includes online policies where buffer size adaptation can be included as part of the policy.

REFERENCES