On the Necessary Conditions for Transmitting Correlated Sources over a Multiple Access Channel

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***Abstract*—We study the lossy communication of correlated sources over a multiple access channel (MAC). In particular, we provide a new set of necessary conditions for the achievability of a distortion pair over a given channel. The necessary conditions are then specialized to the case of bivariate Gaussian sources and doubly symmetric binary sources over a Gaussian multiple access channel. Our results indicate that the new necessary conditions provide the tightest conditions to date in certain cases.**

I. Introduction

We consider the transmission of correlated sources over a multiple access channel (MAC) under individual fidelity criteria. This is one of the fundamental open problems in network information theory, and the set of achievable distortion pairs is unknown in most cases. This problem generalizes another long-standing open problem, namely the multi-terminal lossy source-coding problem, obtained when the underlying MAC consists of two orthogonal finite-capacity error-free links. Despite the lack of a general single-letter characterization for the multi-terminal source coding problem, the optimality of separate source and channel coding can be shown [1] when the underlying MAC is orthogonal. However, due to the lack of a general separation result, the set of achievable distortion pairs is not known even in scenarios for which the corresponding source coding problem can be solved completely; this is the case even for lossless transmission [2], illustrating the difficulty of the problem. However, as it is shown in [3], [4], optimality of separation can emerge also through the availability of a side information at the receiver, conditioned on which the two sources are independent.

In the absence of single-letter necessary and sufficient conditions, the goal is to obtain computable inner and outer bounds. A fairly general joint source-channel coding scheme is introduced in [5] by leveraging hybrid analog-digital coding. This scheme subsumes most other known coding schemes. A novel outer bound is presented in [6] for the Gaussian setting, which uses the fact that the correlation among channel inputs is limited by the correlation available among source sequences. Another bound is proposed in [7] and recently in [8].

Here, we exploit the ideas from [4] in order to obtain new necessary conditions for the achievability of a distortion pair. By providing a side information to the encoders and decoder that enables separation by inducing conditional independence, we are able to obtain computable necessary conditions. Although this idea has previously been used to obtain converse results in multiterminal source coding [9−10], this paper is the first effort that utilizes it in a joint source-channel coding problem to derive novel converse bounds. We show that the proposed technique leads to the tightest known bounds to date in certain scenarios. In particular, we study the transmission of Gaussian and doubly symmetric binary sources (DSBS) over a Gaussian MAC, and provide comparisons of the obtained necessary conditions with those from [6] and [8].

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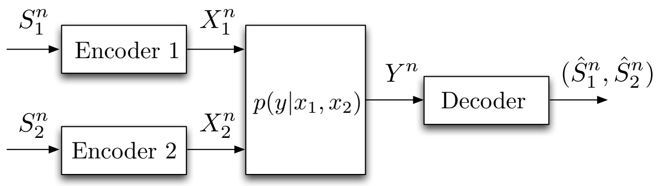

Fig. 1. Correlated sources over a MAC.

In the sequel, we use X for a random variable, x for its realization, and $\mathcal{X}$ to denote its domain. We let $X^n = (X_1, \ldots, X_n)$ be a random vector of length n. We use $\mathbb{E}[X]$ and var(X) for the expected value and variance of X, respectively. The cardinality of $\mathcal{X}$ is denoted by $|\mathcal{X}|$.

II. System Model

We study the transmission of memoryless sources S_1 and S_2 over a memoryless MAC as illustrated in Fig.1. Encoder j observes $S_j^n = (S_{j1}, \ldots, S_{jn})$ and maps it to the channel input $X_j^n = (X_{j1}, \ldots, X_{jn})$, through an encoding function $e_j^{(n)} : \mathcal{S}_j^n \to \mathcal{X}_j^n$, $j = 1, 2$. The channel is characterized by the conditional distribution $p(y|x_1, x_2)$. The decoder observes the channel output Y^n and constructs its estimates for both source sequences, $\hat{S}_1^n$ and $\hat{S}_2^n$, through the decoding functions $g_j^{(n)} : \mathcal{Y}^n \to \hat{\mathcal{S}}_j^n$, $j = 1, 2$. Corresponding average distortion values for the source sequence S_j^n, $j = 1, 2$, are given by

$$\Delta_j^{(n)} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[d_j(S_{ji}, \hat{S}_{ji})], \tag{1}$$

where $d_j(\cdot, \cdot) < \infty$ is the additive distortion measure for source S_j^n. A distortion pair (D_1, D_2) is *achievable* for the source pair (S_1, S_2) and channel $p(y|x_1, x_2)$ if there exists a sequence of encoding and decoding functions $\{e_1^{(n)}, e_2^{(n)}, g_1^{(n)}, g_2^{(n)}\}$ such that

$$\limsup_{n\to\infty} \Delta_j^{(n)} \leq D_j, \quad j=1,2. \tag{2}$$

In the sequel, we will use the following two definitions extensively.

Definition 1. *(Wyner's common information) [11] Wyner's common information between S_1 and S_2 is given as,*

$$C_W(S_1,S_2) = \min_{\substack{p(v|s_1,s_2):\\ S_1-V-S_2}} I(S_1,S_2;V). \tag{3}$$

Definition 2. *(Conditional rate distortion function) [12] Define the minimum average distortion for S_j given Q as [13], [14]:*

$$\mathcal{E}(S_j|Q) = \min_{f:Q\to\hat{S}_j} E[d_j(S_j,f(Q))], \quad j=1,2, \tag{4}$$

Then, the conditional rate distortion function for source S_j when side information Z is shared between the encoder and the decoder is given as,

$$R_{S_j|Z}(D_j) = \min_{\substack{p(u_j|s_j,z):\\ \mathcal{E}(S_j|U_j,Z)\leq D_j}} I(S_j;U_j|Z), \quad j=1,2. \tag{5}$$

III. A Necessary Condition for the Transmission of Correlated Sources over a MAC

We consider in this section the lossy transmission of correlated sources over a MAC, and provide a set of necessary conditions for the achievability of a distortion pair (D_1,D_2). This will be accomplished by providing correlated side information to the encoders and the decoder, conditioned on which the two sources become independent.

A set of necessary conditions for transmitting correlated sources over a MAC is presented in Theorem 1 below.

Theorem 1. *Consider the communication of two correlated sources S_1 and S_2 over a MAC characterized by $p(y|x_1,x_2)$. If a distortion pair (D_1,D_2) is achievable, then for every Z, for which S_1-Z-S_2 form a Markov chain, we have*

$$R_{S_1|Z}(D_1) \leq I(X_1;Y|X_2,Q), \tag{6}$$

$$R_{S_2|Z}(D_2) \leq I(X_2;Y|X_1,Q), \tag{7}$$

$$R_{S_1|Z}(D_1)+R_{S_2|Z}(D_2) \leq I(X_1,X_2;Y|Q), \tag{8}$$

$$R_{S_1S_2}(D_1,D_2) \leq I(X_1,X_2;Y), \tag{9}$$

for some Q for which X_1-Q-X_2 form a Markov chain, where

$$R_{S_1S_2}(D_1,D_2) = \min_{\substack{p(\hat{s}_1,\hat{s}_2|s_1,s_2):\\ \mathbb{E}[d_j(S_j,\hat{S}_j)]\leq D_j,\ j=1,2}} I(S_1,S_2;\hat{S}_1,\hat{S}_2) \tag{10}$$

is the rate distortion function of the joint source (S_1,S_2) with target distortion D_j for source S_j, $j=1,2$.

Proof. For any Z that satisfies the Markov chain condition, we consider the genie-aided setting in which Z^n is provided to both the encoders and the decoder. From [4, Theorem 2], separation is optimal in this setting, and the corresponding necessary and sufficient conditions for the achievability of a distortion pair serve as necessary conditions for the original problem. Conditions (6)-(8) follow from [4], whereas condition (9) follows from the cut-set bound. □

By relaxing conditions (6) and (7), we obtain the following necessary conditions.

Corollary 1. *If a distortion pair (D_1,D_2) is achievable for sources (S_1,S_2), then for every Z that forms a Markov chain S_1-Z-S_2, we have*

$$R_{S_1|Z}(D_1)+R_{S_2|Z}(D_2), \leq I(X_1,X_2;Y|Q) \tag{11}$$

$$R_{S_1S_2}(D_1,D_2) \leq I(X_1,X_2;Y), \tag{12}$$

for some Q such that X_1-Q-X_2.

In the following, we specialize the necessary conditions from Corollary 1 to the transmission of correlated sources over a Gaussian MAC.

Consider a memoryless MAC with additive Gaussian noise:

$$Y = X_1+X_2+N, \tag{13}$$

where N is a standard Gaussian random variable. We impose input power constraints $\frac{1}{n}\sum_{i=1}^n \mathbb{E}[X_{ji}^2] \leq P$, $j=1,2$.

From [15], for the Gaussian MAC, we have

$$I(X_1,X_2;Y|Q) \leq \frac{1}{2}\log(1+\beta_1P+\beta_2P) \tag{14}$$

$$I(X_1,X_2;Y) \leq \frac{1}{2}\log(1+2P+2P\sqrt{(1-\beta_1)(1-\beta_2)}) \tag{15}$$

for some $0\leq\beta_1,\beta_2\leq 1$. From Corollary 1, along with (14) and (15), we obtain the following necessary conditions.

Corollary 2. *If a distortion pair (D_1,D_2) is achievable for sources (S_1,S_2) over a Gaussian MAC characterized in (13), then for every Z that forms a Markov chain S_1-Z-S_2,*

$$R_{S_1|Z}(D_1)+R_{S_2|Z}(D_2) \leq \frac{1}{2}\log(1+\beta_1P+\beta_2P) \tag{16}$$

$$R_{S_1S_2}(D_1,D_2) \leq \frac{1}{2}\log(1+2P+2P\sqrt{(1-\beta_1)(1-\beta_2)}) \tag{17}$$

for some $0\leq\beta_1,\beta_2\leq 1$.

IV. Gaussian Sources over a Gaussian MAC

In this section, we study the transmission of bivariate Gaussian sources over a Gaussian MAC. We compare the necessary conditions in Corollary 2 with the conditions obtained by Lapidoth and Tinguely in [6], and by Lapidoth and Wigger in [8]. Consider a bivariate Gaussian source (S_1,S_2) such that

$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right), \tag{18}$$

transmitted over the memoryless Gaussian MAC in (13), under squared error distortion measures $d_j(S_j,\hat{S}_j) = (S_j-\hat{S}_j)^2$ for $j=1,2$. The common part between S_1 and S_2 in (3) is characterized as follows [16, Proposition 1]. Let V, N_1, and N_2 be standard random variables. We can express S_1, S_2 as,

$$S_i = \sqrt{\rho}V + \sqrt{1-\rho}N_i \text{ for } i=1,2, \tag{19}$$

such that $I(S_1,S_2;V) = \frac{1}{2}\log\frac{1+\rho}{1-\rho}$ and $I(S_1,S_2;V') \geq \frac{1}{2}\log\frac{1+\rho}{1-\rho}$ for all $S_1-V'-S_2$ for which $V'\neq V$.

For this scenario, the tightest known necessary conditions are obtained by Lapidoth and Tinguely in [6, Theorem IV.1]:

$$R_{S_1S_2}(D_1,D_2) \leq \frac{1}{2}\log(1+2P(1+\rho)). \tag{20}$$

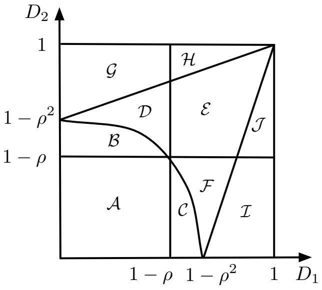

Fig. 2. Partitioned distortion regions for (D_1, D_2).

Another set of necessary conditions is presented by Lapidoth and Wigger in [8, Corollary 1.1]. For correlated Gaussian sources, together with (14)-(15), their conditions are obtained as follows.[2]

$$R_{S_1S_2}(D_1,D_2)-\frac{1}{2}\log\frac{1+\rho}{1-\rho} \leq \frac{1}{2}\log(1+\beta_1 P+\beta_2 P) \quad (21)$$

$$R_{S_1S_2}(D_1,D_2) \leq \frac{1}{2}\log(1+2P+2P\sqrt{(1-\beta_1)(1-\beta_2)}) \quad (22)$$

for some $0 \leq \beta_1, \beta_2 \leq 1$.

In the following, we compare the necessary conditions from Corollary 2 with the conditions from (20) and (21)-(22). We note that since comparing the performance of the full set of necessary conditions from Theorem 1 with [8, Theorem 1] is computationally intensive, we focus on Corollary 2 by relaxing the first two conditions in Theorem 1, and compare this subset of conditions with [6, Theorem IV.1] as in [8, Corollary 1.1]. To do so, we let Z in Corollary 2 to be the common part of (S_1, S_2) from (3), i.e., $Z = V$ from (19).

The rate distortion function for source S_i, $i = 1, 2$, with encoder and decoder side information Z is [18]:

$$R_{S_i|Z}(D_i) = \begin{cases} \frac{1}{2}\log\frac{1-\rho}{D_i} & \text{if} \quad 0 < D_i < 1-\rho \\ 0 & \text{if} \quad D_i \geq 1-\rho \end{cases}. \quad (23)$$

Lastly, for the Gaussian source, (10) is as in [6, Theorem III.1].

In the following, we show that there exist (D_1, D_2) values for which Corollary 2 gives the tightest bound compared to (20) and (21)-(22). To do so, we define

$$r_1(\beta_1,\beta_2) \triangleq \frac{1}{2}\log(1+2P+2P\sqrt{(1-\beta_1)(1-\beta_2)}), \quad (24)$$

$$r_2(\beta_1,\beta_2) \triangleq \frac{1}{2}\log(1+\beta_1 P+\beta_2 P), \quad (25)$$

and consider the region

$$\mathcal{R} = \bigcup_{0\leq\beta_1,\beta_2\leq 1}\{(R_1,R_2) : R_1 \leq r_1(\beta_1,\beta_2), R_2 \leq r_2(\beta_1,\beta_2)\}. \quad (26)$$

Then, the necessary conditions in Corollary 2 state that, if a distortion pair is achievable, then

$$\left(R_{S_1S_2}(D_1,D_2), R_{S_1|Z}(D_1)+R_{S_2|Z}(D_2)\right) \in \mathcal{R}. \quad (27)$$

The necessary conditions from (21)-(22), on the other hand, state that, if a distortion pair (D_1, D_2) is achievable, then

$$\left(R_{S_1S_2}(D_1,D_2), R_{S_1S_2}(D_1,D_2)-\frac{1}{2}\log\frac{1+\rho}{1-\rho}\right) \in \mathcal{R}. \quad (28)$$

[2]We note that our expressions differ slightly from (15) in [8]. Please see [17] for the details.

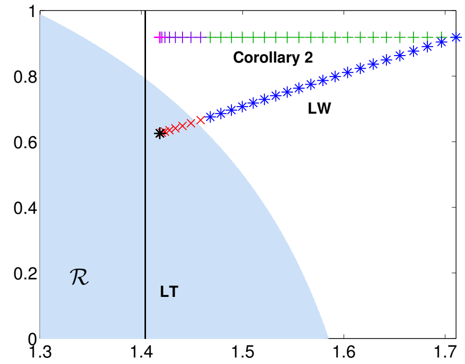

Fig. 3. Comparison of Corollary 2 with LT and LW necessary conditions from (20) and (21)-(22), respectively, for $P = 2$, $\rho = 0.5$, and $D_1 = 0.14$.

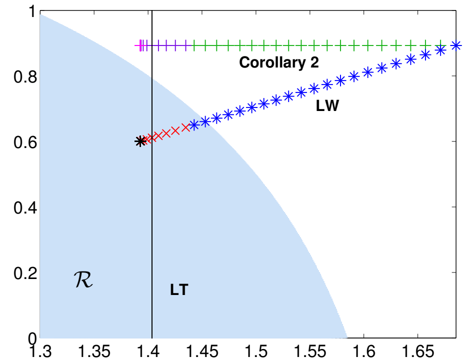

Fig. 4. Comparison of Corollary 2 with LT and LW necessary conditions from (20) and (21)-(22), for $P = 2$, $\rho = 0.5$, and $D_1 = 0.145$.

In the following, we let $\rho = 0.5$ and $P = 2$. We partition the set of all distortion pairs (D_1, D_2), $0 \leq D_1, D_2 \leq 1$, as in Fig. 2, and define the following pairs:

$$g(D_1,D_2) \triangleq \left(R_{S_1S_2}(D_1,D_2), R_{S_1|Z}(D_1)+R_{S_2|Z}(D_2)\right), \quad (29)$$

$$l(D_1,D_2) \triangleq \left(R_{S_1S_2}(D_1,D_2), R_{S_1S_2}(D_1,D_2)-\frac{1}{2}\log\frac{1+\rho}{1-\rho}\right). \quad (30)$$

Consider first Region $\mathcal{B}$ in Fig. 2, for which $D_1 \leq 1-\rho$ and $1-\rho \leq D_2 \leq \frac{1-\rho^2-D_1}{1-D_1}$. We let $D_1 = 0.14 < 1-\rho$. For a (D_1, D_2) pair in Region $\mathcal{B}$, i.e., $D_1 = 0.14$ and $1-\rho \leq D_2 \leq \frac{1-\rho^2-D_1}{1-D_1}$, from (23) we have

$$g(D_1,D_2) = \left(\frac{1}{2}\log\frac{1-\rho^2}{D_1D_2}, \frac{1}{2}\log\frac{1-\rho}{D_1}\right). \quad (31)$$

The $g(D_1, D_2)$ pairs obtained for increasing D_2 values within Region $\mathcal{B}$ are illustrated as points with green "+" sign in Fig. 3. On the other hand, the region $\mathcal{R}$ from (26) is the region shaded in blue in the same figure. Whenever a green point from (31) falls outside the blue region $\mathcal{R}$, we conclude that the corresponding distortion pair (D_1, D_2) is not achievable, according to Corollary 2. We also evaluate

$$l(D_1,D_2) = \left(\frac{1}{2}\log\frac{1-\rho^2}{D_1D_2}, \frac{1}{2}\log\frac{(1-\rho)^2}{D_1D_2}\right) \quad (32)$$

for points $(0.14, D_2)$ in Region $\mathcal{B}$. The points corresponding to $l(D_1, D_2)$ for different D_2 values are shown with a dark blue "*" marking in Fig. 3. Accordingly, whenever such a

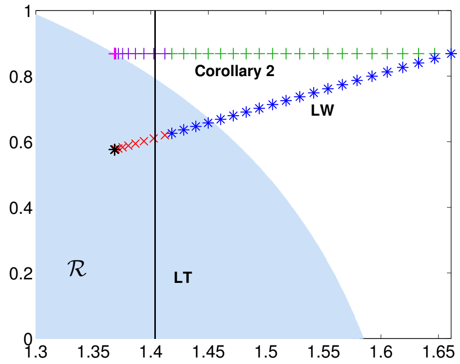

Fig. 5. Comparison of Corollary 2 with LT and LW necessary conditions from (20) and (21)-(22), for $P = 2$, $\rho = 0.5$, and $D_1 = 0.15$.

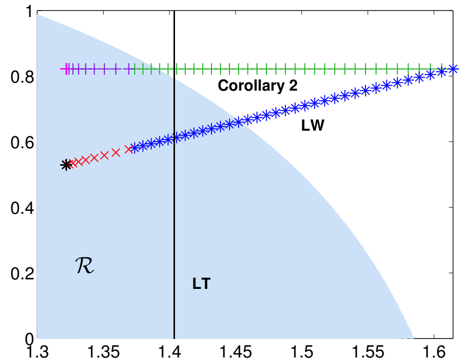

Fig. 6. Comparison of Corollary 2 with LT and LW necessary conditions from (20) and (21)-(22), for $P = 2$, $\rho = 0.5$, and $D_1 = 0.16$.

point from (32) is not contained within region $\mathcal{R}$ from (26), the corresponding (D_1, D_2) pair is not achievable, according to the LW conditions in (21)-(22). We next consider (D_1, D_2) pairs from Region $\mathcal{D}$. We plot in Fig. 3 the values obtained for $D_1 = 0.14$ and D_2 varying from $\frac{1-\rho^2-D_1}{1-D_1}$ to $1-\rho^2+\rho^2 D_1$, with a purple "+" sign for $g(D_1, D_2)$ and with a red "x" marking for $l(D_1, D_2)$. Finally, we consider Region $\mathcal{G}$. We plot in Fig. 3 the values obtained for $1-\rho^2+\rho^2 D_1 \leq D_2 \leq 1$, with a pink "+" sign for $g(D_1, D_2)$ and with a black "*" marking for $l(D_1, D_2)$. These points coincide as $g(D_1, D_2)$ and $l(D_1, D_2)$ depend only on D_1 in this case. The points sharing the same value on the horizontal axis correspond to the same (D_1, D_2) pair, as the first terms of both (29)-(30) are equal. Lastly, we illustrate the right-hand side (RHS) of (20) with a straight line. The points on the RHS of this line correspond to (D_1, D_2) pairs that are not achievable due to (20). In Figs. 4-6, we plot the corresponding regions for $D_1 \in \{0.145, 0.15, 0.16\}$, by keeping the remaining parameters fixed.

In order to compare Corollary 2 with the LT bound from (20) and LW conditions from (21)-(22), we investigate the (D_1, D_2) pairs that cannot be achieved by Corollary 2, (20), and (21)-(22), respectively, in Figs. 3-6. We observe from Fig. 3 that none of the considered (D_1, D_2) pairs is achievable when $D_1 = 0.14$, according to Corollary 2 and the LT bound, whereas some (D_1, D_2) pairs satisfy the LW conditions, as several points labeled with "x" fall into region $\mathcal{R}$. From Fig. 4, we observe that when $D_1 = 0.145$, some (D_1, D_2) pairs from Regions $\mathcal{G}$ and $\mathcal{D}$ (from Fig. 2) satisfy both LT and LW conditions, but not Corollary 2, as some pink and purple points marked with "+" are on the left-hand side (LHS) of the straight line, but outside $\mathcal{R}$. Fig. 5 shows similar results for $D_1 = 0.15$. In Fig. 6, we find that there exist (D_1, D_2) pairs in Region $\mathcal{B}$ that satisfy LT and LW conditions but not Corollary 2 when $D_1 = 0.16$, as several green points marked with "+" are on the LHS of the straight line but outside $\mathcal{R}$. We then conclude that there exist (D_1, D_2) pairs for which Corollary 2 provides tighter conditions than both the LT and LW conditions in Regions $\mathcal{G}$, $\mathcal{D}$, and $\mathcal{B}$, and, by symmetry, in Regions $\mathcal{I}$, $\mathcal{F}$, and $\mathcal{C}$.

By comparing the LHS of Corollary 2 with (21)-(22), and using the fact that the region defined by the RHS of both conditions is the same, we find that Corollary 2 is at least as tight as LW conditions in all regions but $\mathcal{E}$, $\mathcal{D}$, and $\mathcal{F}$. For space considerations, the details are provided in [17]. We remark that this does not necessarily mean that Corollary 2 is strictly tighter in any of these regions, since the necessary conditions involve also the RHS of (16)-(17) and (21)-(22), and they can be used to claim the impossibility of achieving certain distortion pairs based on the relative value of the rate distortion functions with respect to the rate region characterized by the RHS. It is possible that, even though the LHS of Corollary 2 is lower than the LHS of the LW bound, either both or none of the necessary conditions may be satisfied, leading exactly to the same conclusion regarding the achievability of the corresponding distortion pair. On the other hand, while we have numerically shown the existence of cases in which Corollary 2 provides strictly tighter bounds than LW conditions, we have not come across a case in which the opposite holds, that is, LW conditions show that a certain distortion pair is not achievable, while Corollary 2 holds.

V. Binary Sources over a Gaussian MAC

In this section, we consider the transmission of a doubly symmetric binary source (DSBS) over a Gaussian MAC. Consider a DSBS with joint distribution

$$p(S_1\!=\!s_1, S_2\!=\!s_2) = \frac{1-\alpha}{2}(1-|s_1-s_2|) + \frac{\alpha}{2}|s_1-s_2|, \quad (33)$$

a memoryless Gaussian MAC from (13), and Hamming distortion $d_j(S_j, \hat{S}_j) = |S_j - \hat{S}_j|$ where $\hat{\mathcal{S}}_j = \mathcal{S}_j = \{0, 1\}$ for $j = 1, 2$.

For the conditions in Corollary 2, we choose the variable Z as illustrated in Fig. 7(a). Then the joint distribution for (S_i, Z) is as given in Fig. 7(b) for $i = 1, 2$. Note that Z forms a Z-channel both with S_1 and S_2 while satisfying $S_1 - Z - S_2$. Using the conditional rate-distortion function for the Z-channel setting from [19], one can evaluate Corollary 2.

We compare Corollary 2 first with the set of necessary conditions obtained from [6, Remark IV.1],

$$R_{S_1 S_2}(D_1, D_2) \leq \frac{1}{2}\log(1 + 2P(1 + \rho_{max})), \quad (34)$$

where $R_{S_1 S_2}(D_1, D_2)$ is as in [20, Theorem 2], and ρ_{max} is the Hirschfield-Gebelin-Rényi maximal correlation for DSBS given by [21]:

$$\rho_{max} = \sqrt{2(\alpha^2 + (1-\alpha)^2) - 1}. \quad (35)$$

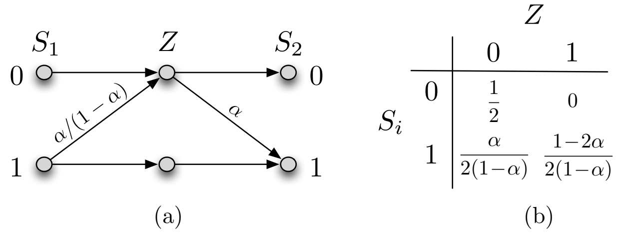

Fig. 7. (a) Z-channel structure. (b) $p(S_i, Z)$ for $i = 1, 2$.

We next obtain the Lapidoth-Wigger necessary conditions from [8, Corollary 1.1] by using (14)-(15),

$$R_{S_1S_2}(D_1,D_2)-1-h(\alpha)+2h(\theta)\leq\frac{1}{2}\log(1+\beta_1P+\beta_2P) \quad (36)$$

$$R_{S_1S_2}(D_1,D_2)\leq\frac{1}{2}\log(1+2P+2P\sqrt{(1-\beta_1)(1-\beta_2)}) \quad (37)$$

for some $0 \leq \beta_1, \beta_2 \leq 1$, where $\theta = (1/2)(1-\sqrt{1-2\alpha})$ and $h(\lambda) = -\lambda\log\lambda - (1-\lambda)\log(1-\lambda)$ is the binary entropy function, and $C_W(S_1, S_2)$ from (3) is as in [11].

The last set of necessary conditions we consider is obtained from [8, Theorem 1] by relaxing (9a) and (9b), letting $W \leftarrow Z$, where Z is as defined in Fig. 7, and using (14)-(15),

$$R_{S_1S_2}(D_1,D_2)-1+\frac{\alpha}{1-\alpha}h(\alpha)\leq\frac{1}{2}\log(1+\beta_1P+\beta_2P), \quad (38)$$

$$R_{S_1S_2}(D_1,D_2)\leq\frac{1}{2}\log(1+2P+2P\sqrt{(1-\beta_1)(1-\beta_2)}), \quad (39)$$

for some $0 \leq \beta_1, \beta_2 \leq 1$. In Fig. 8, we compare the necessary conditions from Corollary 2 with the LT necessary conditions from (34), and the LW conditions from (36)-(37) and (38)-(39) for the setting $D_1 = 0.25$, and $0 \leq D_2 \leq \frac{\alpha}{2(1-\alpha)}$. The green "+" signs are for the points obtained from Corollary 2, whereas blue "*" markings are for the points from (36)-(37) and red "x" markings are for (38)-(39). The straight line represents (34). Fig. 8 shows that there exist (D_1, D_2) pairs for which all conditions except Corollary 2 are satisfied, suggesting Corollary 2 to be tighter under those settings.

VI. Conclusion

We have identified a set of necessary conditions for the lossy transmission of correlated sources over a multiple access channel. We have shown that these conditions provide, in certain cases, the tightest known necessary conditions for the Gaussian setting, i.e., Gaussian sources transmitted over a Gaussian MAC, as well as for a DSBS over a Gaussian MAC. Current and future directions include investigating the necessary conditions for other multi-terminal scenarios.

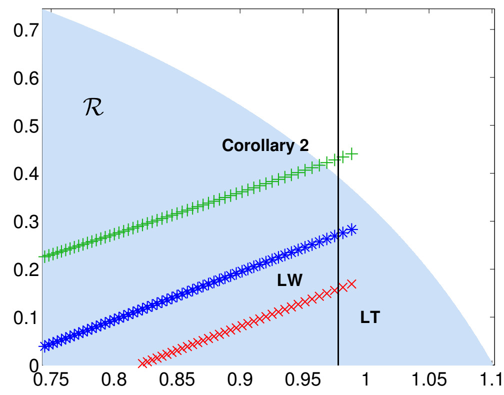

Fig. 8. Comparison of the necessary conditions from Corollary 2 with LT bound from (34) and LW conditions from (36)-(37) and (38)-(39) for the DSBS. $P = 0.9$, $\alpha = 0.2$, $D_1 = 0.25$, and $0 \leq D_2 \leq \frac{\alpha}{2(1-\alpha)}$.

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