Selective Interference Alignment for MIMO Femtocell Networks

Basak Guler and Aylin Yener
Electrical Engineering Department
The Pennsylvania State University, University Park, PA 16801
bxg215@psu.edu *yener@ee.psu.edu*

***Abstract*—An interference limited multitier multiuser MIMO cellular uplink is considered. Specifically, an interference management scheme is proposed where interference from subsets of macrocell users is aligned at the femtocell base stations in order to ensure acceptable service for the femtocell users. The scheme employs interference alignment (IA) at each femtocell base station (FBS), to the set of macrocell users (MU) that are causing the high interference specifically at that FBS, and hence is termed *selective* IA. The proposed IA algorithm determines the interference subspaces at each FBS and precoders for each MU in a distributed fashion. Numerical results demonstrate the performance advantage of selective IA.**

I. INTRODUCTION

Femtocells are small base stations designed mainly for indoor use, to provide high data rates for next generation wireless cellular networks [1]. They are low cost plug and play devices purchased by the subscribers, providing coverage to a small area where they are installed [2]. Femtocell users (FU) utilize the internet backhaul, which reduces the load on the macrocell network enabling the resources to be allocated to the truly mobile users. Another reason for femtocells becoming popular among the wireless operators is that they require no infrastructure, which reduces the maintenance costs. It is preferred for the femtocells to share the frequency band with the existing macrocell network, as the licensed band is highly populated, and frequency is a scarce resource. This fact, combined with the ad hoc nature of femtocells, make cross-tier interference management challenging, and render centralized solutions less than practical. In this paper, we consider the interference management problem, where all femto and macrocell users transmit in the same band, concentrating on the uplink interference caused by the macrocell users (MU) at the femtocell base stations (FBS). This may be destructive when the MU is far from the macrocell base station (MBS) and close to the FBS, thereby transmitting with high power.

As a means of effective interference management, we propose to utilize interference alignment (IA) [3] for aligning the signals from the MUs that are causing high interference at multiple FBS simultaneously. In this two-tier system, just like in single tier systems [3-6], the signals of the interferers (MUs) are restricted to a lower dimensional subspace received at each FBS. This in turn allows the FBS to use fewer receive antennas for canceling the macrocell interference, and to utilize the remaining degrees of freedom for improving the performance of the FUs. However, unlike the single-tier systems, spatial dimensions must be allocated in the best possible way to deal with macro and femtocell interference together. This is the main issue considered in this paper. The proposed solution includes judicious selection of the macrocell interferers to align at multiple FBSs, and identification of the subspace in which cross-tier interference signals would be aligned followed by a distributed algorithm to identify the precoders needed at the selected interferers. Numerical results are then presented to demonstrate the advantage of the proposed approach termed *selective* interference alignment.

The remainder of the paper is organized as follows: We summarize related work in Section II. In Section III, the system model is introduced. In Section IV, we propose the MU selection for IA. In Section V, we present the distributed IA algorithm and its convergence. Section VI provides the relevant discussion and is followed by numerical results in Section VII. The paper is concluded in Section VIII. The notation used is as follows: Lower (upper) bold case letters for vectors (matrices), $\mathbf{A}^\dagger$ for the pseudo-inverse of matrix $\mathbf{A}$. $\mathbf{A}^H$ is used for the Hermitian transpose, and $\otimes$ for the Kronecker product. Finally, $tr(\mathbf{A})$ represents the trace of matrix $\mathbf{A}$, and $|\mathcal{S}|$ denotes the cardinality of the set $\mathcal{S}$.

II. RELATED WORK

IA is proved to achieve the maximum number of degrees of freedom in a K user interference channel [3] by aligning the interfering signals in a lower dimensional subspace at multiple receivers simultaneously. Precoders that can achieve exact IA are known only for the 3 user interference channels [3]. In order to develop methods for IA as the number of users increase, several distributed algorithms are proposed, including minimizing the leakage interference [4], alternating minimization over unitary precoders and receive subspaces [6], maximizing the SINR [4], or minimizing MSE [5]. These algorithms are developed specifically for K user interference channels, in which each transmitter has an intended receiver, and the remaining transmitters are considered as interferers for that receiver. As an example, the minimum leakage interference/max SINR algorithms proposed in [4] use channel reciprocity and iterate between the receivers and transmitters at each step by reversing the communication direction [11] in order to minimize the leaked interference/maximize the SINR of the intended signal, respectively.

In [10], we proposed a method for using IA for eliminating macrocell interference, while meeting the QoS requirements of the MUs, in terms of minimum SINR constraints at the MBS. MUs that were causing high interference to a group of FBSs were aligned at all the FBSs in this group. This approach could result in less than desirable performance when the set of MUs that are causing high interference at each FBS is distinct. In order to address this problem, in this paper we choose the MUs that are causing the highest interference at each FBS and

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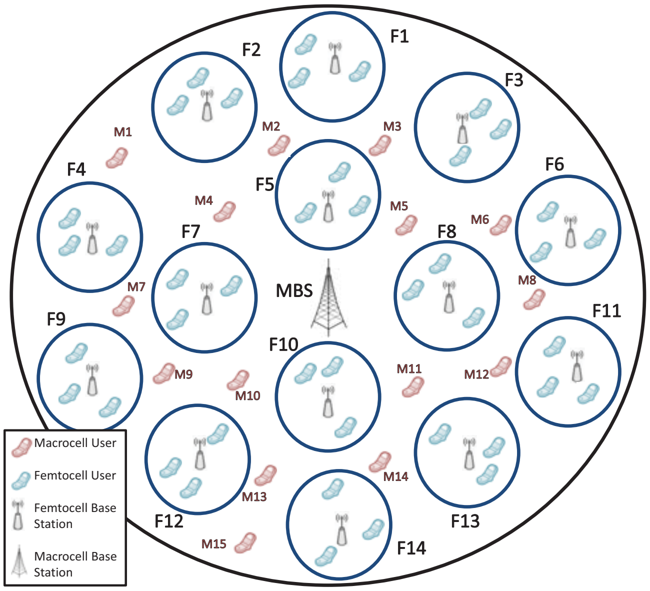

Fig. 1. System model for one MBS and multiple FBSs.

apply IA only among these users. Therefore, the set of aligned users at each FBS will be different from one another and will be specific to that FBS and its interfering MUs.

A user selection method for a K user interference channel was considered in [9]. However, as minimum leakage algorithm in [9] determines both precoder and decoders, and since we are applying IA for macrocell users to align them at the FBSs, the approach would require to design the decoders of the macrocell users at the FBSs, which is not acceptable due the excessive computational load it would cause at the FBSs, the internet backhaul and the macrocell network. Hence a new approach is needed for the two-tier network at hand. We shall describe this in the sequel.

III. System Model

The cellular network considered in this paper is the uplink of a co-existing macrocell-femtocell network with a single MBS at the center and multiple FBSs distributed over the macrocell coverage area, as shown in Fig. 1[1]. Each mobile user has N_t transmit antennas. The MBS and the FBSs have N_o and N_f receive antennas, respectively. The number of MUs is denoted by M and the number of FUs at the f^{th} FBS is denoted by U_f. The received signal at the k^{th} FBS is given as:

$$\mathbf{y}_k = \sum_{i=1}^{U_k} \sqrt{p_{ki}}\mathbf{H}_{kk}^{i}\mathbf{W}_{ki}\mathbf{s}_{ki} + \sum_{\substack{f=1\\ f\neq k}}^{F} \sum_{u=1}^{U_f} \sqrt{p_{fu}}\mathbf{H}_{kf}^{u}\mathbf{W}_{fu}\mathbf{s}_{fu}$$
$$+ \sum_{m=1}^{M} \sqrt{p_{om}}\mathbf{H}_{ko}^{m}\mathbf{W}_{om}\mathbf{s}_{om} + \mathbf{n}_k \qquad (1)$$

where $\mathbf{W}_{fu}$ denotes the $(N_t \times d)$ precoding matrix of the u^{th} user of the f^{th} femtocell, and $\mathbf{W}_{om}$ represents the $(N_t \times d)$ precoding matrix of the m^{th} MU. The number of message bits transmitted from each mobile user is denoted by d. We assume the same number of bits are transmitted from each

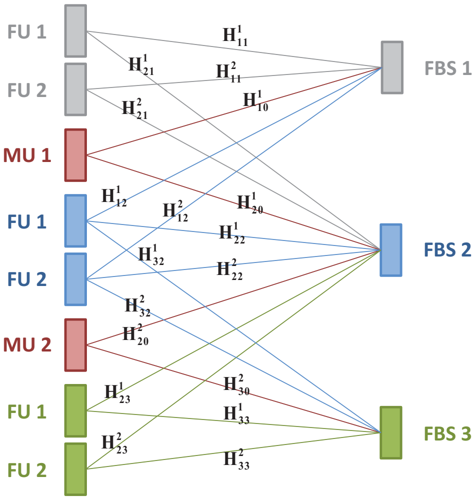

Fig. 2. Channel model for 3 FBSs, with 2 FUs in each femtocell, and 2 MUs.

mobile user to simplify the analysis, noting that the results obtained in this paper can be extended to the case in which different number of bits are transmitted from each user. $\mathbf{s}_{fu}$ is the $(d\times 1)$ message signal of the u^{th} user of the f^{th} femtocell, and $\mathbf{s}_{om}$ represents the $(d \times 1)$ message signal of the m^{th} MU. $\mathbf{H}_{ko}^{m}$ represents the channel from the m^{th} MU to the k^{th} FBS, and $\mathbf{H}_{kf}^{u}$ is the channel from the u^{th} user of the f^{th} femtocell to the k^{th} FBS. The noise vector at the k^{th} FBS is denoted by $\mathbf{n}_k$, which consists of independent zero mean Gaussian random variables with $E\{\mathbf{n}_k {\mathbf{n}_k}^H\} = \sigma^2\mathbf{I}$. Each element of the message signals $\mathbf{s}_{fu}$ and $\mathbf{s}_{om}$ is chosen from $\{+1, -1\}$ randomly with equal probability for $u = 1, \ldots, U_f$, $f = 1, \ldots, F$, and $m = 1, \ldots, M$. The power transmitted from the f^{th} user of the u^{th} femtocell is p_{fu} , and the transmit power of the j^{th} MU is p_{oj}. The precoding matrices of all mobile users satisfy $tr(\mathbf{W}_{oj}^H\mathbf{W}_{oj}) = tr(\mathbf{W}_{ku}^H\mathbf{W}_{ku}) = 1, \forall k \in \{1, F\}, u \in \{1, U_k\}, j \in \{1, M\}$. An example system model with 3 FBSs and 2 MUs is depicted in Fig. 2, where the first MU is assumed to be causing high interference at FBSs 1 and 2, and the second MU is assumed to be causing high interference at FBSs 2 and 3.

IV. Macrocell User Selection

The maximum number of users that can be aligned at each FBS is limited by the number of antennas, i.e., the spatial dimensions available for IA. Consequently, aligning all MUs at all FBSs simultaneously is not feasible for typical user loads. Instead of attempting to align the entire set of MUs at every FBS simultaneously, we propose a user selection algorithm in which only the *dominant* MUs, i.e., the MUs that cause the highest interference, are aligned at each FBS. The user selection process starts with determining the MU that is causing the highest interference at each FBS. Then, we define the set of dominant MUs at FBS k as $\mathcal{S}_k = \{\mathcal{S}_k^1, \mathcal{S}_k^2, \ldots, \mathcal{S}_k^{|S_k|}\}$ where

$$\mathcal{S}_k^1 = \underset{j\in\{1,\ldots,M\}}{\arg\max}\; tr((\mathbf{H}_{ko}^{j})^H\mathbf{H}_{ko}^{j})$$
$$\mathcal{S}_k^i = \{j \in \{1, \ldots, M\} : tr((\mathbf{H}_{ko}^{j})^H\mathbf{H}_{ko}^{j}) \geq$$
$$\tau tr((\mathbf{H}_{ko}^{S_k^1})^H\mathbf{H}_{ko}^{S_k^1}), |S_k| \leq n\}, \quad \forall i \neq 1 \qquad (2)$$

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[1] We treat inter-macrocell interference as noise and concentrate on one macrocell.

where $\tau \in [0,1]$ is a constant that we use to compare the interference caused by each MU compared to the highest MU interferer at that FBS. Specifically, it is the set of MUs whose interference is at least a fraction τ of the interference caused from the highest MU interferer. The set of FBSs at which the i^{th} MU will be aligned is given as:

$$\mathcal{A}_i = \{j \in \{1, \ldots, F\} : i \in \mathcal{S}_j\} \qquad \forall i = 1, \ldots, M \tag{3}$$

Next, the IA algorithm presented in the next section is applied to the set of MUs at each FBS. Note that, the set of aligned users for each FBS can be different from one another, due to the channel conditions and the location of the users and base stations, as well as the number of MUs to be aligned at each FBS. This approach allows the FBS to adapt to different conditions. For example, when the number of dominant MU interferers is low, it can allocate its resources mainly for its own users, achieving high data rates by multiplexing, and when the number of high interferers increases, it can devote a necessary amount of its resources for aligning these MUs and to prevent signal degradation for its own users. It is important to note that the choice of τ as well as the maximum number of aligned users, n depends on the available resources, feasibility requirements and system conditions, and the selection decision is made at a central unit that handles the system, and communicated to the FBSs via the backhaul.

V. DISTRIBUTED INTERFERENCE ALIGNMENT

A. Formulation

In order to align the dominant MU interferers, we define the interference subspaces at each FBS such that the received signals from the selected MUs at each FBS will span the subspace specific to that FBS. For this purpose, we define matrices $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_F$ such that the columns of these matrices define the basis for the subspaces for the aligned interference at each receiver. That is, each column of $\mathbf{H}_{ko}^j \mathbf{W}_{oj}$ can be written as a linear combination of the columns of $\mathbf{V}_k$, $\forall j \in \mathcal{S}_k$, and $\forall k \in \{1, F\}$. The IA condition requires that the received signals from the MU set defined for each FBS to span the same subspace, which is given as:

$$\begin{aligned}
\mathbf{H}_{1o}^j \mathbf{W}_{oj} &\prec \mathbf{V}_1 \qquad \forall j \in \mathcal{S}_1 \\
\mathbf{H}_{2o}^j \mathbf{W}_{oj} &\prec \mathbf{V}_2 \qquad \forall j \in \mathcal{S}_2 \\
&\vdots \\
\mathbf{H}_{Fo}^j \mathbf{W}_{oj} &\prec \mathbf{V}_F \qquad \forall j \in \mathcal{S}_F
\end{aligned} \tag{4}$$

where $\mathbf{X} \prec \mathbf{Y}$ denotes that the column space of $\mathbf{Y}$ spans that of $\mathbf{X}$. Let us denote the i^{th} column of $\mathbf{V}_k$ by $\mathbf{v}_k^i$, or equivalently $\mathbf{V}_k = [\mathbf{v}_k^1 \mathbf{v}_k^2 \ldots \mathbf{v}_k^d]$ and the i^{th} column of $\mathbf{W}_{oj}$ as $\mathbf{w}_{oj}^i$, i.e., $\mathbf{W}_{oj} = [\mathbf{w}_{oj}^1 \mathbf{w}_{oj}^2 \ldots \mathbf{w}_{oj}^d]$ Then the conditions for IA [8] at FBSs $k = 1, \ldots F$ can be described as follows:

$$\begin{aligned}
\mathbf{H}_{ko}^j \mathbf{w}_{oj}^1 &= \alpha_{kj}^1 \mathbf{v}_k^1 + \beta_{kj}^1 \mathbf{v}_k^2 + \ldots + \theta_{kj}^1 \mathbf{v}_k^d, \ \forall j \in \mathcal{S}_k \\
\mathbf{H}_{ko}^j \mathbf{w}_{oj}^2 &= \alpha_{kj}^2 \mathbf{v}_k^1 + \beta_{kj}^2 \mathbf{v}_k^2 + \ldots + \theta_{kj}^2 \mathbf{v}_k^d, \ \forall j \in \mathcal{S}_k \\
&\vdots \\
\mathbf{H}_{ko}^j \mathbf{w}_{oj}^d &= \alpha_{kj}^d \mathbf{v}_k^1 + \beta_{kj}^d \mathbf{v}_k^2 + \ldots + \theta_{kj}^d \mathbf{v}_k^d, \ \forall j \in \mathcal{S}_k
\end{aligned} \tag{5}$$

where α_{kj}^i is a constant and the given equations require that all the MUs that are in the “interference set” of a FBS span the same column space, i.e., the received signals from those specific MUs are represented by a linear combination of the subspace basis vectors, scaled by different coefficients. The conditions in (5) can be represented in terms of linear matrix equations as follows:

$$\widetilde{\mathbf{H}}_{kj} \mathbf{w}_{oj} = \widetilde{\mathbf{A}}_{kj} \mathbf{v}_k, \quad \forall j \in \mathcal{S}_k \tag{6}$$

where $\mathbf{w}_{oj} = \begin{bmatrix}(\mathbf{w}_{oj}^1)^T & (\mathbf{w}_{oj}^2)^T & \ldots & (\mathbf{w}_{oj}^d)^T\end{bmatrix}^T \ \forall j \in \mathcal{S}_k$, and $\mathbf{v}_k = \begin{bmatrix}(\mathbf{v}_k^1)^T & (\mathbf{v}_k^2)^T & \ldots & (\mathbf{v}_k^d)^T\end{bmatrix}^T$. $\widetilde{\mathbf{H}}_{kj}$ is a block diagonal matrix with d blocks of $\mathbf{H}_{ko}^j$:

$$\widetilde{\mathbf{H}}_{kj} = \begin{bmatrix} \mathbf{H}_{ko}^j & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{ko}^j & \ldots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{H}_{ko}^j \end{bmatrix} \ \forall j \in \mathcal{S}_k \tag{7}$$

The coefficient matrices $\widetilde{\mathbf{A}}_{kj}$ for $\forall j \in \mathcal{S}_k$ are:

$$\widetilde{\mathbf{A}}_{kj} = \begin{bmatrix} \alpha_{kj}^1 & \beta_{kj}^1 & \ldots & \theta_{kj}^1 \\ \alpha_{kj}^2 & \beta_{kj}^2 & \ldots & \theta_{kj}^2 \\ \vdots & & \ddots & \vdots \\ \alpha_{kj}^d & \beta_{kj}^d & \ldots & \theta_{kj}^d \end{bmatrix} \otimes \mathbf{I}_{N_r \times N_r} \tag{8}$$

where $\mathbf{I}_{N_r \times N_r}$ denotes the $(N_r \times N_r)$ identity matrix, and

$$\mathbf{A}_{kj} = \begin{bmatrix} \alpha_{kj}^1 & \beta_{kj}^1 & \ldots & \theta_{kj}^1 \\ \alpha_{kj}^2 & \beta_{kj}^2 & \ldots & \theta_{kj}^2 \\ \vdots & & \ddots & \vdots \\ \alpha_{kj}^d & \beta_{kj}^d & \ldots & \theta_{kj}^d \end{bmatrix} \tag{9}$$

When we follow this procedure for each receiver, the necessary conditions for IA at F FBSs can be represented as:

$$\widetilde{\mathbf{H}}_{kj} \mathbf{w}_{oj} = \widetilde{\mathbf{A}}_{kj} \mathbf{v}_k, \quad \forall j \in \mathcal{S}_k, \quad k \in \{1, F\} \tag{10}$$

B. Algorithm

The proposed distributed algorithm is as follows:

1) Initialize the matrices $\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_F$ and $\mathbf{A}_{kj}$ $\forall k = 1, \ldots, F$ and $\forall j = 1, \ldots, M$.

2) Determine the precoding vectors $\mathbf{w}_{o1}, \mathbf{w}_{o2}, \ldots, \mathbf{w}_{oM}$ as:

$$\begin{aligned}
\mathbf{w}_{oj} = \arg\min_{\mathbf{w}_{oj}} \quad &\sum_{k \in \mathcal{A}_j} \|\widetilde{\mathbf{H}}_{kj} \mathbf{w}_{oj} - \widetilde{\mathbf{A}}_{kj} \mathbf{v}_k\|^2 \\
s.t. \quad &tr((\mathbf{w}_{oj})^H \mathbf{w}_{oj}) = 1
\end{aligned} \tag{11}$$

where the equality constraint is used to guarantee that the transmit power of each MU is $tr((\sqrt{p_{oj}} \mathbf{w}_{oj})^H \sqrt{p_{oj}} \mathbf{w}_{oj}) = p_{oj}$.

3) Construct the precoding matrices $\mathbf{W}_{o1}, \mathbf{W}_{o2}, \ldots, \mathbf{W}_{oM}$ using the precoding vectors from Step 2).

4) Fix the precoding matrices and determine the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_F$ as follows:

$$\mathbf{v}_k = \arg\min_{\mathbf{v}_k} \quad \sum_{j \in \mathcal{S}_k} \left\|\widetilde{\mathbf{H}}_{kj} \mathbf{w}_{oj} - \widetilde{\mathbf{A}}_{kj} \mathbf{v}_k\right\|^2 \tag{12}$$

5) Determine the coefficients $\mathbf{A}_{kj}$ for $k = 1, \ldots, F$ and $j = 1, \ldots, M$ according to the following procedure. For a given $\mathbf{H}_{ko}^j, \mathbf{W}_{oj}$ and $\mathbf{V}_k$, construct the following equation:

$$(\mathbf{H}_{ko}^j \mathbf{w}_{oj}^k)^T = \underbrace{\begin{bmatrix} \alpha_{kj}^i & \beta_{kj}^i & \ldots & \theta_{kj}^i \end{bmatrix}}_{(\mathbf{a}_{kj}^i)^T} (\mathbf{V}_k)^T \tag{13}$$

Then $\mathbf{a}_{kj}^i = \mathbf{V}_k^\dagger \mathbf{H}_{ko}^j \mathbf{w}_{oj}^i$, and $\mathbf{A}_{kj}^T = \begin{bmatrix} \mathbf{a}_{kj}^1 & \mathbf{a}_{kj}^2 & \ldots & \mathbf{a}_{kj}^d \end{bmatrix}$ where $\mathbf{V}_k^\dagger$ denotes the pseudo-inverse of the matrix $\mathbf{V}_k$:

$$\mathbf{V}_k^\dagger = (\mathbf{V}_k^H \mathbf{V}_k)^{-1} \mathbf{V}_k^H \tag{14}$$

6) Iterate from Step 2) to Step 5) until convergence.

This algorithm is distributed in the sense that, each MU needs to know the channel gains from itself to the receivers it is interfering, so that it can apply the algorithm and determine its own precoding vector. Each MU also needs to know $\widetilde{\mathbf{A}}_{kj}\mathbf{v}_k$, $\forall k \in \mathcal{A}_j$ (for the j^{th} MU).

C. Details of Step 2) and 4)

The solution for problem (11) in Step 2) is constructed by relaxing the equality constraint into an inequality constraint, which turns problem (11) into a quadratically constrained quadratic problem (QCQP) as:

$$\mathbf{w}_{oj} = \arg\min_{\mathbf{w}_{oj}} \sum_{k\in\mathcal{A}_j} \|\widetilde{\mathbf{H}}_{kj}\mathbf{w}_{oj} - \widetilde{\mathbf{A}}_{kj}\mathbf{v}_k\|^2 \\ s.t. \quad tr((\mathbf{w}_{oj})^H\mathbf{w}_{oj}) \leq 1 \tag{15}$$

Then the KKT conditions for (15) can be formulated as:

$$\begin{aligned} &\textit{Stationarity:} \sum_{k\in\mathcal{A}_j} \widetilde{\mathbf{H}}_{kj}\mathbf{w}^*_{oj} + \lambda_j\mathbf{w}^*_{oj} - \sum_{k\in\mathcal{A}_j} \widetilde{\mathbf{H}}^H_{kj}\widetilde{\mathbf{A}}_{kj}\mathbf{v}_k = 0 \\ &\textit{Complementary Slackness:} \quad \lambda_j((\mathbf{w}^*_{oj})^H\mathbf{w}^*_{oj} - 1) = 0 \\ &\textit{Dual feasibility:} \quad \lambda_j \geq 0 \\ &\textit{Primary feasibility:} \quad (\mathbf{w}^*_{oj})^H\mathbf{w}^*_{oj} \leq 1 \end{aligned} \tag{16}$$

where $\mathbf{w}^*_{oj}$ denotes the optimal $\mathbf{w}_{oj}$. Using (16), the solution for the optimal $\mathbf{w}_{oj}$ for $j = 1, \ldots, M$ can be found as:

$$\mathbf{w}^*_{oj} = (\sum_{k\in\mathcal{A}_j} (\widetilde{\mathbf{H}}_{kj})^H\widetilde{\mathbf{H}}_{kj} + \lambda_j\mathbf{I})^{-1} \sum_{k\in\mathcal{A}_j} \widetilde{\mathbf{H}}^H_{kj}\widetilde{\mathbf{A}}_{kj}\mathbf{v}_k \tag{17}$$

where λ_j is calculated such that $\text{tr}((\mathbf{w}_{oj})^H\mathbf{w}_{oj}) = 1$. The optimality condition for the unconstrained problem (12) is:

$$\sum_{j\in\mathcal{S}_k} \widetilde{\mathbf{A}}^H_{kj}\widetilde{\mathbf{A}}_{kj}\mathbf{v}^*_k - \sum_{j\in\mathcal{S}_k} \widetilde{\mathbf{A}}^H_{kj}\widetilde{\mathbf{H}}_{kj}\mathbf{w}_{oj} = 0 \tag{18}$$

where $\mathbf{v}^*_k$ is the optimal $\mathbf{v}_k$. Using this condition, we obtain the optimal $\mathbf{v}_k$ for $k = 1, \ldots, F$ as:

$$\mathbf{v}^*_k = (\sum_{j\in\mathcal{S}_k} \widetilde{\mathbf{A}}^H_{kj}\widetilde{\mathbf{A}}_{kj})^{-1}(\sum_{j\in\mathcal{S}_k} \widetilde{\mathbf{A}}^H_{kj}\widetilde{\mathbf{H}}_{kj}\mathbf{w}_{oj}) \tag{19}$$

Using (19), we can determine the matrices $\mathbf{V}_1, \ldots, \mathbf{V}_F$.

D. Convergence Analysis

In this section, we provide the convergence analysis for the proposed algorithm in Section IV. We define the total leaked interference from all MUs and FBSs as:

$$C = \sum_{k=1}^{F}\sum_{j\in\mathcal{S}_k} \|\widetilde{\mathbf{H}}_{kj}\mathbf{w}_{oj} - \widetilde{\mathbf{A}}_{kj}\mathbf{v}_k\|^2 \tag{20}$$

$$= \sum_{j=1}^{M}\sum_{k\in\mathcal{A}_j} \|\widetilde{\mathbf{H}}_{kj}\mathbf{w}_{oj} - \widetilde{\mathbf{A}}_{kj}\mathbf{v}_k\|^2 \tag{21}$$

When $\mathbf{v}_1, \ldots, \mathbf{v}_F$ are fixed, $\mathbf{w}_{o1}, \ldots, \mathbf{w}_{oM}$ are determined according to (11), which decreases the value of (21), which then decreases the value of C. Similarly, when $\mathbf{w}_{o1}, \ldots, \mathbf{w}_{oM}$ are fixed, we determine $\mathbf{v}_1, \ldots, \mathbf{v}_F$ using (12), which decreases the value of (20), from which we see that C has also decreased. Thus we conclude that C is decreased after each iteration, and since C is bounded below by zero, the algorithm converges. However, due to the non convex nature of the problem, this algorithm does not guarantee convergence to the global optimum, and may end up at a local optimum.

VI. DISCUSSION

It is at this point useful to discuss the reasons for being able to use the distributed approach in Section V. If we did not specify the subspaces for each FBS, and still wanted to use IA, then the MUs would have to share information with other MUs (such as channel and coefficient information), which is not preferred due to the excessive load to mobile users and privacy issues. If we did not want to share information between users, we would have to use a centralized algorithm as in [10], but since in the present scheme we are considering the whole femtocell network instead of a group of FBSs, using a centralized method would require sending the channel information and the information about the IA sets for each FBS before each transmission to a centralized processor, and the centralized processor would solve the IA problem with excessive amounts of data, and send back the determined precoders to the MUs, over the MBS-MUs link. Instead, here, essentially, we have divided this single problem into multiple problems that can be solved at each FBS locally, in parallel with other FBSs which saves from this overhead. In the proposed scheme, each FBS needs to share channel and coefficient information only with the MUs in its IA set $\mathcal{S}_k$ to create its IA subspace. The MUs only use the information about the subspaces of the FBSs in their IA set $\mathcal{A}_j$ and their channels to those FBSs. The sole constraint for the IA problem is the transmit power constraint, and no other assumptions are made on the precoders/subspaces, which renders the problem easy to relax into different types, such as an SDP problem, and add additional constraints such as minimum SINR requirements, which can be employed for improving the QoS of MUs [10], if so desired.

VII. SIMULATION RESULTS

Simulations are performed to compare the performance of the FUs for two tiered network models with single MBS and multiple FBSs for two scenarios. First scenario is for a dense urban model, with 18 FBSs distributed over a macrocell area with $155m$ radius, and the MUs are placed at the cell edges of the FBSs for analyzing the effect of the cell-edge MUs, as given in Fig. 3. Each hexagonal cell in Fig. 3. denotes a femtocell and is approximated by a circular area with a radius of $30m$. The second scenario has a MBS with a coverage radius of 500m and 14 FBSs distributed randomly, each with a coverage radius of $30m$, this model is similar to the one in Fig. 1. In both systems, each femtocell has 3 randomly distributed FUs, the mobile users have 4 transmit antennas, and the FBSs have 7 receive antennas.

Selective IA is compared with the IA with femtocell grouping scheme, in which the macrocell area is divided into smaller areas to form a femtocell group as in [10]. The MUs and the FBSs within this group cooperate to apply IA to the received signals of the MUs at the FBSs, similar to the model defined in [10]. The radius for a femtocell group is assumed to be $75m$, as presented in Fig. 3. A single bit stream is transmitted from each user. Noise power is assumed to be -110dB. We consider Rayleigh fading channels with the indoor/outdoor path loss modeled according to the ITU-R channel model [7] specifications. We have empirically chosen $\tau = 0.1$ and $n = 4$ for MU selection.

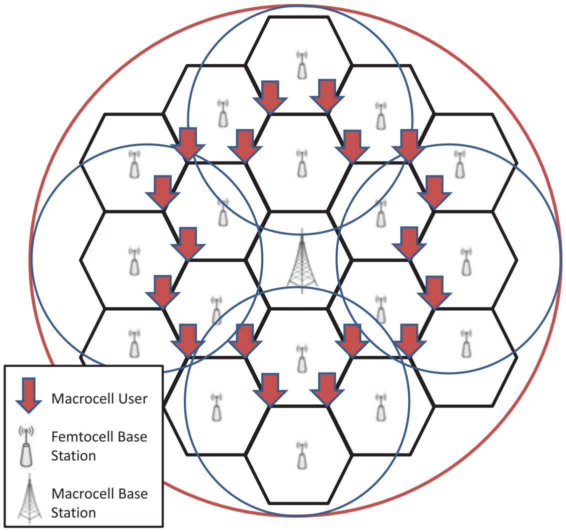

Fig. 3. Model for the dense urban femtocell network.

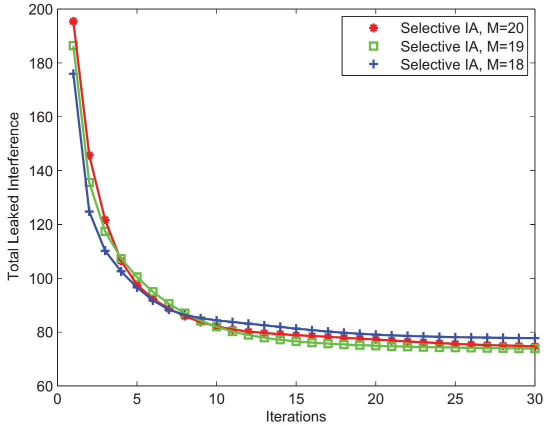

Fig. 4. Convergence results of the Selective-IA Algorithm for M=18, 19 and 20 macrocell users.

For the first system, the convergence of the distributed algorithm is demonstrated in Fig. 4, and the performance of the FUs with respect to the transmit powers of 18 MUs in Fig. 5. For the second system, we compare the average BERs of the FUs for the selective and femtocell grouping schemes as given in Fig. 6, with a maximum transmit power of 1W for each user. The selective and femtocell grouping IA schemes were also compared to a case where the base stations and the MUs for IA are selected randomly. The simulation results confirm the intuition that the judicious selection of MU for IA is beneficial as compared to these two schemes.

VIII. Conclusion

In this paper, we have considered a scheme that is applicable to a tiered network where the interferers from one tier are distributed over the whole network. We have focused on a two-tier system with coexisting femtocells and a macrocell, and proposed using user selection at the FBSs combined with a distributed IA algorithm to eliminate the destructive uplink macrocell interference at the FBSs. The proposed algorithm is constructed in such a way that is specifically applicable to the tiered network and that it mitigates the problems that may arise from using a centralized IA algorithm, due to backhaul limitations and the excessive load caused on the network. Future work includes considering QoS requirements of the MUs, and designing robust systems with reduced complexity, and incomplete/estimated channel state information.

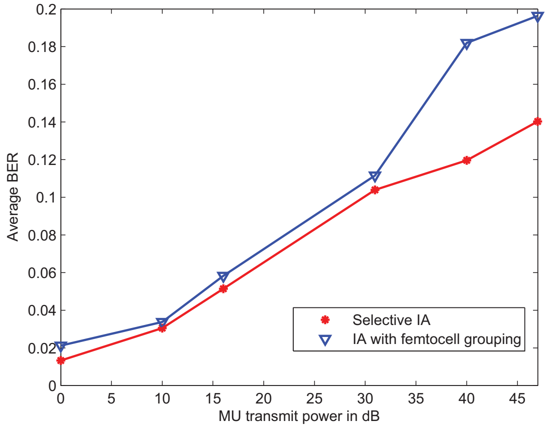

Fig. 5. Average BER of the FUs compared to the MU transmit power.

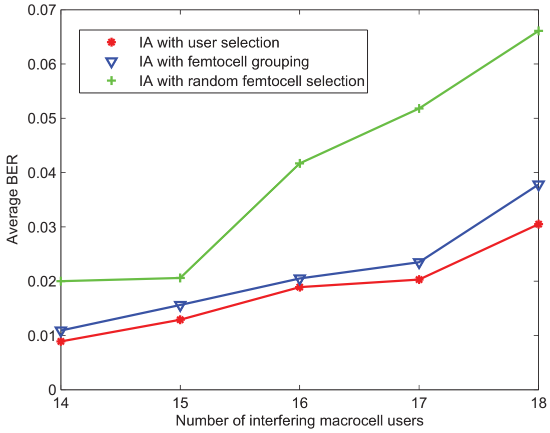

Fig. 6. Average BER of the femtocell users wrt. number of macrocell interferers.

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